# REGIONAL TRAINING ON MACRO-FISCAL FORECASTING

**ARUSHA, AUGUST 8 – 18, 2016**

## Day 1: Monday August 8

<table>
<thead>
<tr>
<th>Time</th>
<th>Subject</th>
<th>Resource Person</th>
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</thead>
<tbody>
<tr>
<td>08:30 – 09:00</td>
<td>Registration and Introduction</td>
<td></td>
</tr>
<tr>
<td>09:00 – 09:30</td>
<td>Quiz</td>
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</tr>
<tr>
<td>09:30 – 10:15</td>
<td>L1. Overview of Macroeconomic Forecasting</td>
<td>Fazeer Rahim</td>
</tr>
<tr>
<td>10:15 – 10:45</td>
<td>Coffee break and Group Photo</td>
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<tr>
<td>10:45 – 12:30</td>
<td>W1. Introduction to Forecasting Using EViews</td>
<td>Fazeer Rahim</td>
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<tr>
<td>12:30 – 13:30</td>
<td>Lunch Break</td>
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<tr>
<td>13:30 – 14:30</td>
<td>L2. Properties of Time Series Data I: Stationarity, Box Jenkins ARIMA Models</td>
<td>Heloisa Marone</td>
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<tr>
<td>15:00 – 15:30</td>
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<tr>
<td>14:30 – 17:00</td>
<td>W2. Estimating and Forecasting ARIMA Models using EViews</td>
<td>Heloisa Marone</td>
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## Day 2: Tuesday August 9

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<tr>
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<tbody>
<tr>
<td>08:30 – 09:30</td>
<td>L3. Properties of Time Series Data II: Nonstationarity and Unit Roots</td>
<td>Heloisa Marone</td>
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<tr>
<td>09:30 – 10:00</td>
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<tr>
<td>10:00 – 12:30</td>
<td>W3. Detecting Nonstationary Time Series in Practice</td>
<td>Heloisa Marone</td>
</tr>
<tr>
<td>12:30 – 13:30</td>
<td>Lunch Break</td>
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<tr>
<td>14:30 – 15:00</td>
<td>Coffee break</td>
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<tr>
<td>15:30 – 16:30</td>
<td>W4. Forecasting using Single Equation Estimation Methods and ECM Models</td>
<td>Fazeer Rahim</td>
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### Day 3: Wednesday August 10

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<tbody>
<tr>
<td>08:30 – 10:15</td>
<td>L5. Vector Autoregression (VAR), Structural VAR Models, Impulse Response Functions (IRFs)</td>
<td>Fazeer Rahim</td>
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<tr>
<td>10:15 – 10:45</td>
<td>Coffee break</td>
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<tr>
<td>10:45 – 12:30</td>
<td>W5. VAR and IRF Application: UK Money Demand—Part I</td>
<td>Fazeer Rahim</td>
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<tr>
<td>13:00 – 14:00</td>
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<tr>
<td>14:00 – 14:45</td>
<td>L6. Cointegration II: Johansen Methodology</td>
<td>Heloisa Marone</td>
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<td>16:00 – 16:30</td>
<td>Coffee break</td>
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<tr>
<td>16:30 – 17:30</td>
<td>W6. UK Money Demand—Part II</td>
<td>Heloisa Marone</td>
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### Day 4: Thursday August 11

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<tbody>
<tr>
<td>08:30 – 09:30</td>
<td>L7. Vector Error Correction Models: Formulation, Hypothesis Testing, and Forecasting</td>
<td>Heloisa Marone</td>
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<tr>
<td>09:30 – 10:00</td>
<td>Coffee break</td>
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<tr>
<td>10:00 – 12:30</td>
<td>W7. Forecasting using a VECM</td>
<td>Heloisa Marone</td>
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<tr>
<td>13:00 – 14:00</td>
<td>Lunch Break</td>
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<tr>
<td>14:00 – 17:00</td>
<td>Preparation for participants’ presentation</td>
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### Day 5: Friday August 12

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<tbody>
<tr>
<td>09:00 – 10:00</td>
<td>L8. Evaluating regression models</td>
<td>Phyllis Resnick</td>
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<tr>
<td>10:30 – 11:00</td>
<td>Coffee break</td>
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<tr>
<td>11:00 – 12:30</td>
<td>W8. Evaluating regression models</td>
<td>Phyllis Resnick</td>
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<td>12:30 – 13:30</td>
<td>Lunch Break</td>
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<td>13:30 – 16:30</td>
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### Day 6: Monday August 15

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<tr>
<td>09:00 – 10:15</td>
<td>L9. Elements of Revenue Forecasting I: Macroeconomic Assumptions and the Effective Tax Rate Approach</td>
<td>Fazeer Rahim</td>
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<tr>
<td>10:30 – 11:00</td>
<td>Coffee break</td>
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<tr>
<td>10:45 – 12:30</td>
<td>W9. Revenue Forecast</td>
<td>Fazeer Rahim</td>
</tr>
<tr>
<td>12:30 – 13:30</td>
<td>Lunch Break</td>
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<tr>
<td>14:30 – 17:00</td>
<td>W10. Revenue Forecast II</td>
<td>Phyllis Resnick</td>
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### Day 7: Tuesday August 16

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<tr>
<td>09:00 – 12:30</td>
<td>L11. Expenditure Forecasting</td>
<td>Phyllis Resnick</td>
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<tr>
<td>12:30 – 13:30</td>
<td>Lunch Break</td>
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</tr>
<tr>
<td>13:30 – 16:30</td>
<td>W11. Expenditure Forecasting</td>
<td>Phyllis Resnick</td>
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### Day 8: Wednesday August 17

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<tbody>
<tr>
<td>09:00 – 10:30</td>
<td>L12. Overall balance, net lending/borrowing and debt, financing the budget and monetary policy</td>
<td>Phyllis Resnick</td>
</tr>
<tr>
<td>10:30 – 11:00</td>
<td>Coffee break</td>
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</tr>
<tr>
<td>11:00 – 12:30</td>
<td>W13. Completing the budget forecast and projecting debt</td>
<td>Phyllis Resnick</td>
</tr>
<tr>
<td>12:30 – 13:30</td>
<td>Lunch Break</td>
<td></td>
</tr>
<tr>
<td>14:30 – 15:00</td>
<td>Coffee break</td>
<td></td>
</tr>
<tr>
<td>15:00 – 17:30</td>
<td>W14. Analysis of Fiscal Stance</td>
<td>Fazeer Rahim</td>
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### Day 9: Thursday August 18

<table>
<thead>
<tr>
<th>Time</th>
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<th>Resource Person</th>
</tr>
</thead>
<tbody>
<tr>
<td>09:00 – 11:00</td>
<td>Group presentations and wrap up</td>
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<tr>
<td>11:00 – 11:30</td>
<td>Coffee break</td>
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<tr>
<td>11:30 – 13:00</td>
<td>Concluding session</td>
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<tr>
<td>13:00 – 14:00</td>
<td>Lunch Break</td>
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Workshop on Macro-Fiscal Forecasting
Arusha, August 8 – 18, 2016

L1. Overview of Macroeconomic Forecasting

Fazeer Rahim (IMF East AFRITAC)

Outline

1. Preliminaries
2. Quick overview of forecasting methods
3. Evaluation of forecasts
4. Week 1 – Format of training

This training material is the property of the International Monetary Fund (IMF) and is intended for use in IMF Institute courses. Any reuse requires the permission of the IMF Institute.
Preliminaries

Choices for Forecasting

- What is the variable of interest? For example, GDP growth or inflation.

- What is the forecast horizon? For example, one quarter or one year.

- Would you like point or interval forecasts? For example, GDP growth next year will be 2% or GDP growth will be between 1% and 3% with a probability of 80%? Latter probably more useful for policy makers, since point forecasts are by definition random variables subject to error.

Preliminaries

Example: point vs interval forecast

Uganda: 5-year projection of Uganda's debt to GDP ratio
Preliminaries

Forecast sources

- **Informal methods**: Surveys, polls, personal judgment
  - DGP (data generation process) is not modeled explicitly

- **Econometric models**: non-structural and structural
  - Attempt to approximate the true DGP using an econometric model.

- **Combination forecasts**: combination of informal and econometric approaches

It is advisable to employ several forecasting methods and pool the obtained forecasts

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Preliminaries

Why are forecasts often “wrong”?

- Classic sampling error (the probability that the statistical technique detects the true DGP is zero). And even if it does detect the true DGP, its error term is unpredictable

- The forecasting model is misspecified

- Lucas critique: economic policy and private agents react to existing forecasts (good or bad). In other words, the outcome may depend on the forecast itself (“rational expectations”)
Preliminaries
Forecasting approaches

A. Judgmental approach

B. Time Series Methods
   - Naïve (Random walk) model
   - Deterministic and stochastic trend models
   - ARMA, ARIMA (Box-Jenkins) models
   - VAR/VECM models
   - Structural econometric models

Overview of forecasting
Judgmental approach

• Short-run intelligence on recent past, current period and near future

• Can use more data than will work in an econometric model

• Frequently used to forecast exogenous variables

• But hard to explain and justify

• Hard to determine why errors occurred and avoid next time
Overview

Time series models

1. Naïve (random walk)
2. Deterministic trend
3. Box-Jenkins
4. Vector Autoregression
5. Structural econometric models

A. Naïve (Random walk) model

- Forecast is equal to current value
  \[ y_t = y_{t-1} + u_t \quad u_t \sim NI(0, \sigma^2) \]
  \[ E_t[y_{t+1}] = y_t + E_t[u_{t+1}] \]
  \[ = y_t \]

- Often used as a benchmark
- But it is a “Black box” (no economic theory)
Overview
B. Deterministic trend models

• Fitting linear, quadratic or higher trends
• Example (linear):

\[ y_t = \alpha_1 + \alpha_2 t + u_t \quad u_t \sim \text{NI}(0, \sigma^2) \]
\[ E_t[y_{t+1}] = \alpha_1 + \alpha_2 (t + 1) + E_t[u_{t+1}] \]
\[ = \alpha_1 + \alpha_2 (t + 1) \]

• Another possible benchmark
• It is easy and fast and there is no exogenous variable to forecast
• But it is a “Black box” (no economic theory)

Overview
C. Box-Jenkins (ARIMA) models

• Univariate without exogenous variables: forecast only one variable
• Idea: capture the dynamics of a time series, that is how
  • Past values determine current value (autoregressive component)
  • Past shocks to the variable determine its current value (moving average component)
• Example: ARMA(2,1)
Overview
C. Box-Jenkins models – pros and cons

• Pros:
  • Easy and fast to implement and estimate
  • No exogenous variable to forecast
  • But remains a “Black box” (no economic theory)

Overview
D. Vector Auto-Regression (VAR) models

• Multiple equations (i.e., multiple variables to forecast) without (typically) exogenous variables
• Depends on
  • the past values of all the variables included in the VAR model
• Example: VAR(2,2)

\[ y_t = \alpha_{11}y_{t-1} + \alpha_{12}y_{t-2} + \beta_{11}x_{t-1} + \beta_{12}x_{t-2} + u_{1t} \quad u_{1t} \sim NI(0, \sigma_i^2) \]
\[ x_t = \alpha_{21}y_{t-1} + \alpha_{22}y_{t-2} + \beta_{21}x_{t-1} + \beta_{22}x_{t-2} + u_{2t} \quad u_{2t} \sim NI(0, \sigma_j^2) \]
### Overview
D. Vector Auto-Regression (VAR) models

- Forecast multiple variables simultaneously

\[
E_t[y_{t+1}] = \alpha_{11}y_t + \alpha_{12}y_{t-1} + \beta_{11}x_t + \beta_{12}x_{t-1} + E_t[u_{t+1}]
\]

\[
= \alpha_{11}y_t + \alpha_{12}y_{t-1} + \beta_{11}x_t + \beta_{12}x_{t-1}
\]

\[
E_t[x_{t+1}] = \alpha_{21}y_t + \alpha_{22}y_{t-1} + \beta_{21}x_t + \beta_{22}x_{t-1} + E_t[u_{2,t+1}]
\]

\[
= \alpha_{21}y_t + \alpha_{22}y_{t-1} + \beta_{21}x_t + \beta_{22}x_{t-1}
\]

- Simple econometric methods (e.g., OLS) and inference tests can be used to identify the optimal number of lags and to estimate the coefficients.

### Overview
D. Vector Autoregressions - pros and cons

- Pros:
  - Easy and fast
  - No exogenous variable to forecast

- Cons:
  - Need a large sample size, especially if one wants to predict more than 4 or 5 variables
  - Still a “Black Box” (no economic theory)
## Overview

### Structural econometric models

- Built using macroeconomic theory, estimated using econometric methods

### Example:
- Economic theory tells us that exports depend on competitiveness and global demand
- Using past observations, estimate short run and long run elasticities of exports with respect to:
  - global demand
  - competitiveness
- Forecast global demand and competitiveness.
- Use these forecasts and estimated elasticities to forecast exports.

## Overview

### Structural models - Pros and Cons

**Pro**
- Can accompany figures with an economic “story”
- Allows for simulations – useful for policy analysis

**Cons**
- Require forecast of exogenous variables
- Can be very subjective
Evaluation of Forecasts

- Forecast errors
- Forecast comparison
- Forecast pooling

Sources of forecast errors

Sources

a) Inappropriately formulated model

b) Inaccurately estimated model

c) Data generating process (DGP) has changed over time in unanticipated ways
Suppose you have data from time 0 up to time T.

In-sample analysis
- Estimate model using all available data up to time T, then compare the model's fitted values to the actual realizations

Out-of-sample analysis
- True out-of-sample. Estimate model until T, construct a forecast for T+1. Wait until T+1, record the forecast error and reestimate model to make a forecast for T+2. Do this for a number of periods until there is sufficient forecast errors recorded to allow for an assessment of the model
- Pseudo (simulated) out-of-sample. Split the dataset into two periods (0, T_0), and (T_0+1, T).
  Estimate model over (0, T_0), then use model to forecast over (T_0+1, T)
  Advantage: no need to wait

2 options: Dynamic forecast vs. Static Forecast. Dynamic: At time T_0+2, we use the forecast at time T_0+1 to predict T_0+2. Static: At time T_0+2, we use the actual realizations at time T_0+1 to forecast T_0+2

Evaluation of Forecasts
Forecast comparison

- Mean Absolute Error (MAE): mean of absolute values of forecast errors
- Mean Square Error (MSE): mean of squared forecast errors
- Root Mean Square Error (RMSE): square root of MSE
- Relative RMSE: ratio of the forecast RMSE to the RMSE of a benchmark forecast. A relative RMSE smaller than one indicates that the forecast is better than the benchmark. When the benchmark is a random walk forecast, the relative RMSE is called Theil coefficient
Evaluation of forecasts
Example: comparing one semester ahead OECD and random walk forecasts for French output gap

OECD forecast Mean Absolute Error = (0.228+0.742+1.636+0.543+0.101+0.193+0.069+1.499)/8 = 0.626
OECD forecast Mean Square Error = (0.052+0.551+2.676+0.295+0.010+0.037+0.005+2.247)/8 = 0.734
OECD forecast Root Mean Square Error = (0.734)^{1/2} = 0.857

Random Walk forecast Root Mean Square Error = 1.058

Theil coefficient = 0.857/1.058 = 0.810

<table>
<thead>
<tr>
<th>Actual figure</th>
<th>OECD Forecast</th>
<th>Forecast Error</th>
<th>Absolute Error</th>
<th>Random Walk forecast</th>
<th>Forecast Error</th>
<th>Absolute Error</th>
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<tbody>
<tr>
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<td>0.669</td>
<td>0.33</td>
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5. Evaluation of Forecasts

Forecast Pooling

• Rather than selecting one out of many alternative forecasts for the same variable, we could combine them. The combined (pooled) forecast is

\[ E_t[y_{t+1}] = \alpha_1 E_t[y_{t+1}^1] + \alpha_2 E_t[y_{t+1}^2] + \ldots + \alpha_N E_t[y_{t+1}^N] \]

• For example, we could pool forecasts based on the random walk model (the winner of a forecasting competition) and the structural model with economic story

• There are many methods to select the combination weights \( \alpha_1, \ldots, \alpha_N \). In practice, equal weights, \( \alpha_1 = 1/N, \ldots, \alpha_N = 1/N \), works well, possibly after dropping the worst forecasts from the set under analysis.

Focus of macroeconomic part of the course

• Focus is on a suite of time series models that can be used as to forecast key variables of interest for central banks and finance ministries.

• Elements we will cover
  • Understanding time-series properties
  • Model design and economic theory
  • Model evaluation and diagnostic testing
  • Forecasting using your preferred model

• Monday – Thursday: Lectures followed by a practical session

• Friday: Case study and presentation

• Macro part of the course provides a good basis to complete the online course in MFx: Macroeconomic Forecasting. Next offering will start September 14. Course lasts 7 weeks, and require 6-7 hours of study weekly.
Case Study
Modeling and Forecasting US Saving Rate

Purpose: to develop and improve your modeling and forecasting techniques by analyzing a current issue with a real dataset.

Case study: Forecasting U.S. Saving Rate

Data: Eviews MF.WF1 file, pagefile(“USA_CY")

Thank you
Outline of the lecture

1. The concept of cointegration
2. Spurious regressions: regressing variables that are not cointegrated
3. Testing for cointegration
4. Estimating the cointegrating vector
5. Cointegration and Error-Correction Models
Part I. Cointegration and the concept of equilibrium relationship

The concept of cointegration

Some theories imply the existence of a long-run equilibrium relationship between economic variables:

- Friedman’s permanent income hypothesis: there is a link between consumption and disposable income
- Purchasing power parity: prices developments in two countries, and changes in nominal exchange rate are such that the real exchange rate is constant

In most cases, these theories explain the joint behavior of non-stationary variables over the long-run
Introduction

Cointegration is the framework for testing, and estimating:

- the equilibrium relationship between non-stationary variables
- the short term dynamic behavior of these variables around the long-run equilibrium path

The permanent income hypothesis

Consider Friedman’s permanent income hypothesis:

\[ C_t^* = \beta Y_t^p \]

- \( C_t^* \) is the desired or equilibrium level of consumption
- \( Y_t^p \) is permanent disposable income

At each point in time:

- *actual* consumption \( C_t \) may differ from \( C_t^* \)
- *actual* disposable income \( Y_t \) may differ from \( Y_t^p \)

\[ C_t = \beta Y_t + z_t \]

If the theory holds, \( z_t \) (transitory consumption) must be random and stationary.
The permanent income hypothesis

Disposable income and consumption for selected countries

Purchasing power parity

According purchasing power parity (PPP):

\[ E_t \frac{P_t^*}{P_t} = 1 \]

- \( P_t^* \) and \( P_t \) are the price levels in the foreign and domestic economy (generally, they are non-stationary)
- \( E_t \) is the nominal exchange rate, units of domestic currency per one unit of foreign currency (generally, non-stationary)

At each period, the actual exchange rate may differ from \( \frac{P_t}{P_t^*} \) (taking logs)

\[ e_t + (P_t^* - P_t) = z_t \]

- As far as the deviation \( z_t \) are random and stationary, PPP holds
Cointegration is the exception not the norm

In general, a generic linear combination of non-stationary variables is non-stationary

Cointegration defined

The variables $Y_t$ and $X_{1,t},...,X_{k,t}$ are cointegrated if:

- $Y_t, X_{1,t},...,X_{k,t}$ are all I(1), and
- there exists a vector of non zero coefficient $[1, -\beta_1, ..., -\beta_k]$ such that the linear combination

$$z_t = Y_t - \beta_1 X_{1,t} - ... - \beta_k X_{k,t}$$

is a stationary ARMA($p,q$) process.

It what follows we assume that there is only ONE co-integrating vector: that is, the variables $X_1,...,X_k$ are not cointegrated themselves, and they are not influenced by $Y$ (no feedback).
Cointegration and integration of same order

Another way in which $Y_t$ and $X_t$ are not cointegrated is if:

- the variables are integrated of different order ($Y_t$ is I(p), while $X_t$ is I(q))

For example, suppose that $Y_t$ is I(1) and $X_t$ is stationary:

- How can you possibly explain the long-run growth of $Y_t$ (the trend) with a variable that does not grow in the long-run (it actually tends to revert to a fixed constant)?

Part II. Spurious regressions
OLS on I(1) un-related variables

Consider regressing the following non-stationary variables:

\[ y_t = y_{t-1} + \varepsilon_{y,t} \quad \varepsilon_{y,t} \sim N(0,1) \]
\[ x_t = x_{t-1} + \varepsilon_{x,t} \quad \varepsilon_{x,t} \sim N(0,1) \]

Refresher of OLS assumptions

Refresher on Ordinary Least Square assumptions \( y_t = \alpha + \beta x_t + z \)

\[ E[z_t] = 0 \]
\[ \text{Var}(z_t) = \sigma^2 \]
\[ \text{Cov}(z_t z_{t-1}) = 0 \]
\[ \text{Cov}(z_t z_{t-1-j}) = 0 \]

OLS estimates are unbiased and have the minimum variance compared to other estimators.

Implication of above is that error term \( z_t \) is stationary.

But when you regress two non-stationary variables which are not-cointegrated, the error term is not stationary.
Similar reasoning for misspecified models

Consider the following non-stationary variables:

\[ x_t = x_{t-1} + \varepsilon_{x,t} \]
\[ z_t = z_{t-1} + \varepsilon_{z,t} \]
\[ y_t = 1 + 2x_t + 3z_{t-1} + \varepsilon_{y,t} \]

By construction, \( \varepsilon_{y,t} \) and \( \varepsilon_{x,t} \) are not cointegrated. The model is misspecified because an I(1) variable is missing.

Monte Carlo results

The OLS estimates have the same problems as before: the distribution of \( \hat{\beta} \) does not converge to the true value, even with large samples.
OLS and spurious regression

Conclusion: If you are modeling I(1) variables, it is crucial that the variables be cointegrated, otherwise results and tests do not make sense

If the variable are not cointegrated, OLS regressions will provide spurious results

Notice, if the variables are cointegrated, OLS estimates the cointegrating vector very well

Part III. Testing for cointegration
The Engle and Granger 2-Step Approach

Step 1. estimating the cointegration regression by OLS, and obtain the residual $z_t$
Step 2. Apply ADF test on $z_t$

Note: The null hypothesis ($H_0$) of the ADF test is the residual $z_t$ has a unit root. So, for cointegration, we need to have rejected the null hypothesis.

Summary

If the estimated residuals have a unit root, the model is NOT cointegrated.

If we don’t have co-integration:

- we either augment the long-run model by including additional $l(1)$ variables
- or we simply reject our long-run theory/hypothesis, e.g. PPP.
Example: US and Japan consumption

Consider US (left) and Japan (right) log consumption and income

![Graphs showing consumption and income trends for US and Japan over time with LNCUS and LNYUS for US, and LNCJ and LNYJ for Japan.]

We cannot reject the presence of unit root in the residuals for both US and Japan.
Part IV. Estimating the Cointegrating Vector

Estimating the long-run relationship

If \( Y, X_1, \ldots, X_k \) are cointegrated, OLS provides an estimate of the cointegrating vector.

Under cointegration, OLS is “super consistent”:

- estimates of the parameters approach the true values very quickly as the number of observations increases
- the estimated parameters are consistent even if any of \( X_1, \ldots, X_k \) is correlated with the error term (no asymptotic simultaneity bias; however, the bias may be substantial in small samples)
- \( R^2 \) approaches one as the number of observations increases

However, the OLS estimator can be badly biased in small sample due to correlations
Super consistency

Suppose that $Y$ and $X$ are cointegrated; estimate:

$$y_t = \alpha + \beta x_t + z_t \quad \Delta y_t = \beta \Delta x_t + \Delta z_t$$

Inference in cointegrated systems

Now suppose we want to do some standard hypothesis tests using a cointegrated regression, for example: “is $\beta = 1$?”

While OLS produces acceptable parameter estimates, it does not, in general, yield valid standard errors (and hence t-statistics):

- residuals $z_t$ can be serially correlated and correlated with the innovations in the explanatory variables $\epsilon_{x,t}$
- the correlations of $z_t$ with $\epsilon_{x,t}$ can induce substantial bias in small samples
FM and CCR approach

If the residuals of the co-integrating equation \((z_t)\) and the changes in the regressors \((\epsilon_{x,t})\) are correlated:

- the Fully Modified Least Squares (Phillips and Hansen) “FM”
- the Canonical Cointegrating Regressions (Park) “CCR”

obtain asymptotically unbiased and fully efficient estimates of the parameters.

Conclusion: they produce estimates that can be used for hypothesis testing

Broadly speaking, both methods (both available in recent versions of EViews):

- estimate the long run correlation between \(z_t\) and \(\epsilon_{x,t}\)
- adjust \(y_t\) using these estimated long run correlation

use the adjusted \(y_t\) to estimate parameters.

Part V. Cointegration and Error Correction Models
Granger representation theorem

Engle and Granger (1987) show that co-integration implies the existence of an “error correction model” (ECM) of the form:

$$\Delta y_t = c + \alpha (y_{t-1} - \beta x_{t-1}) + \sum_{j=1}^{p} \phi_j \Delta y_{t-j} + \sum_{j=1}^{p} \gamma_j \Delta x_{t-j} + \epsilon_t$$

- $\beta$ expresses the long-run equilibrium relationship
- $\alpha$ expresses the speed of adjustment, or how strongly the past disequilibrium affects changes in $y$
- $p$ is the order of the AR process in $y$ and $x$
- $\phi$ and $\gamma$ are the parameters of the “own” dynamics of $y$ and $x$ (not reflecting disequilibrium conditions)

Idea: Cointegrated variables both wander stochastically, but stay near each other. They adjust to each other’s locations through a process of error correction. The ECM shows this adjustment process.

The adjustment towards equilibrium

The parameter $\alpha$ (also called the factor loading) indicates the speed of adjustment towards equilibrium. Consider

$$\Delta y_t = 0.16 + \alpha (y_{t-1} - 0.8x_{t-1}) + \epsilon_t$$

Suppose that $x$ grows linearly, the system is in equilibrium, but it receives a shock of 1 at $t = 5$. 

![Graphs showing $x$, $y^*$, and $y_t$ for $\alpha = -0.2$ and $\alpha = -0.8$](graphs.png)
Constraints on $\alpha$

In order for the ECM to be well-defined (and for cointegration)

$-2 < \alpha < 0$

- $\alpha < 0$ so that $y$ can go back to equilibrium after a deviation (a positive deviation requires less of an increase in $y$)
- $\alpha > -2$ otherwise there is “overshooting” with oscillations

Estimation of ECM

STEP 1: Establish cointegration

STEP 2: If cointegration is established, estimate ECM

Further checks:

- Does the coefficient on the adjustment process imply convergence to equilibrium?
- Is any non-stationary variable missing? (check this by looking at the residuals of the ECM...are they white noise?)
- Are the residuals of the ECM non-stationary? If so, there may be structural break in the sample. If that is the case, add dummies.
Example of ECM: Deficits

- Example: Interest Rates and Deficits
- Do large Federal budget deficits drive up long-term interest rates?
- y: 10 year treasury bond rate
- x1: the 1 year rate, x2: inflation, x3: the real deficit per capita, x4: the change in real per capita income.

Source: Econometrics A Modern Introduction by M. Murray, 2006

Example of ECM: Deficits

- Augmented Dickey–Fuller tests suggest that the interest rates, inflation, deficit per capita, and level of income per capita are all integrated I(1).
- OLS regression (on the right) is run.
- Augmented Dickey-Fuller test (below) is done on the residual. Result: we reject the null hypothesis of unit root in the residual. Thus, there is cointegration

Augmented Dickey–Fuller Test Equation
Dependent Variable: D(RESID01)
Method: Least Squares
Sample: 1954 1998
Included observations: 45 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>RESID01(-1)</td>
<td>-0.819757</td>
<td>0.150067</td>
<td>-5.29148</td>
<td>0.0000</td>
</tr>
<tr>
<td>C</td>
<td>0.005599</td>
<td>0.052582</td>
<td>0.106474</td>
<td>0.9152</td>
</tr>
</tbody>
</table>
Forecasting

Assume that:
- the ECM model is correctly specified
- the speed of adjustment and equilibrium relationship will hold in the future

We can forecast the impact of a shock to one variable (e.g. one-off fall in permanent income) on the other dependent variable, distinguishing between the adjustment to the long-run relationship and the short-run impact (via the autoregressive components).

Conclusion

- Cointegration is the framework for testing, and estimating the equilibrium relationship between non-stationary variables and the short term dynamic behavior of these variables around the long-run equilibrium path
- Only if variables are cointegrated one can estimate the long-run relationship and the parameters of the ECM representation
- The parameter $\beta$ captures the long-run relationship; $y = \beta x$ only holds in the long-run
- The parameter $\alpha$ captures the short-run dynamic around the equilibrium and must be negative to have cointegration
Thank you
Workshop on Macro-Fiscal Forecasting  
Arusha, August 8 – 18, 2016

L5 – Vector Autoregressions, Structural VAR Models, Impulse Response Function

Fazeer Rahim (IMF East AFRITAC)

Multivariate Time-Series Models

- Definition - VAR
- Stationarity
- Choosing the lag length
- Forecasting using VARs
- Granger causality test
- The impulse response function
- Structural VAR
Definition - Vector Autoregressions (VAR)

What do to when we are not confident that a variable is actually exogenous, a natural extension is to treat each variable symmetrically – i.e., endogenously.

VAR model is an extension of *univariate* autoregression (AR) model to multivariate (Vector) time series data.

VAR model is a multi-equation system where all the variables are treated as *endogenous*, i.e. there is one equation for each variable as the dependent variable.

VAR allows study of *joint dynamic and contemporaneous* relations between variables (autoregressive).
Do we really need VAR?

- Good at capturing co-movements of multiple time series, i.e. allows analysis of system response to different shocks/impacts.
- Do not need to specify which variables are endogenous or exogenous - all are endogenous!
- Allows the value of a variable to depend on more than just its own lags or combinations of white noise terms, so more general than ARMA modelling.
- Provided that there are no contemporaneous terms on the right hand side of the equations, can simply use OLS separately on each equation.
- VAR forecasts are often better than “traditional structural” models.
- Note: In VAR, variables must be of the same frequency!

What do we report in VAR?

- In VAR, we are not interested in coefficients.
- 3 main components:
  1. Granger-causality tests – does X cause movements in Y?
  2. Impulse response (IRFs) – innovations/shocks
  3. Forecasts of the variables
- These are more informative to understanding the relationships among variables than the VAR regression coefficients or the $R^2$ statistics.

Note: When the variables of a VAR are cointegrated, we use a vector error-correction model (VECM) – more on this tomorrow’s lecture.
A VAR(1) with two variables

Consider a one-variable AR(1):
\[ y_t = a + b_1 y_{t-1} + \epsilon_t \]

A VAR(1) for two variables

In matrix form:
\[
\begin{bmatrix}
    y_{1,t} \\
    y_{2,t}
\end{bmatrix} =
\begin{bmatrix}
    c_1 \\
    c_2
\end{bmatrix} +
\begin{bmatrix}
    b_{11} & b_{12} \\
    b_{21} & b_{22}
\end{bmatrix}
\begin{bmatrix}
    y_{1,t-1} \\
    y_{2,t-1}
\end{bmatrix} +
\begin{bmatrix}
    \epsilon_{1,t} \\
    \epsilon_{2,t}
\end{bmatrix}
\]

Or as a set of equations:
\[
y_{1,t} = c_1 + b_{11} \cdot y_{1,t-1} + b_{12} \cdot y_{2,t-1} + \epsilon_{1,t} \]
\[
y_{2,t} = c_2 + b_{21} \cdot y_{1,t-1} + b_{22} \cdot y_{2,t-1} + \epsilon_{2,t}
\]

Extension to many variables and lags

Let \( y_t \) be a vector with the value of \( n \) variables at time \( t \):
\[
y_t = \begin{bmatrix}
    y_{1,t} \\
    y_{2,t} \\
    \vdots \\
    y_{n,t}
\end{bmatrix}
\]

for example: \( y_t = \begin{bmatrix} M_t \\ P_t \\ I_t \\ GDP_t \end{bmatrix} \)

A \( p \)-order vector autoregressive process (VAR) generalizes a one-variable AR(p) process to \( n \) variables

\[
y_t = c + B_1 y_{t-1} + B_2 y_{t-2} + \ldots + B_p y_{t-p} + \epsilon_t
\]

\( c = (n \times 1) \) vector of constants
\( E[\epsilon_t] = 0 \)
\( B_j = (n \times n) \) matrix of coefficients
\( \epsilon_t = (n \times 1) \) vector of white noise innovations
\( E[\epsilon_t, \epsilon_{t+\tau}] = \begin{cases} \Omega & \text{if } t = \tau \\ 0 & \text{otherwise} \end{cases} \)
Estimation of VAR

\[ y_t = c + B_1 y_{t-1} + B_2 y_{t-2} + \ldots + B_p y_{t-p} + \varepsilon_t \]

Since no current values of \( y_t \) appear on the right hand side, the above VAR model can easily be estimated by OLS and able to produce asymptotically desirable estimators.
Stationarity of a VAR: Definition

A $p$-th order VAR is said to be covariance-stationary if:

- the expected value of $y_t$ does not depend on time

\[ E[y_t] = E[y_{t+j}] = \mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_p \end{bmatrix} \]

- The covariance matrix of $y_t$ and $y_{t+j}$ depends on the time lapsed $j$ and not on the reference period $t$

\[ E[(y_t - \mu)(y_{t-j} - \mu)'] = E[(y_{t-j} - \mu)(y_{t-j} - \mu)'] = \Gamma_{-j} \]
\[ \Gamma_j' = \Gamma_{-j} \]

If a VAR process is covariance stationary then all of its $n$ components are stationary.

Example of a stationarity VAR

In a stationary VAR variables eventually stabilize within a band

Plot of $y_{1,t}$ as function of $t$

Plot of $y_{2,t}$ as function of $t$
Example of a non-stationary VAR

If a VAR is non-stationary, variables diverge

Plot of $y_{1,t}$ as function of $t$

Plot of $y_{2,t}$ as function of $t$

Plot of $y_{1,t}$ and $y_{2,t}$ as function of $t$

Example of a non-stationary VAR

If a VAR is non-stationary, (at least one) variable diverges

Plot of $y_{2,t}$ as function of $t$

Plot of $y_{1,t}$ as function of $t$
Stationarity of a VAR: Definition

If a VAR is stationary, then:

\[ \mu = c + B_1\mu + B_2\mu + \ldots + B_p\mu \]

Substituting into the VAR equation:

\[ (y_t - \mu) = c + B_1(y_{t-1} - \mu) + B_2(y_{t-2} - \mu) + \ldots + B_p(y_{t-p} - \mu) + \epsilon_t \]

The vector \( \mu \) contains the “equilibrium” values of each variable:

- Because \( E[\epsilon] = 0 \) and these innovations are independent across time, their effect cancels out over time
- Deviations of each variable from equilibrium reduce over time; each variable eventually converges to its expected value.

Consequences of stationarity for estimation

If a VAR is stationary, one can estimate the parameters of the process:

- the rows of the \( B \) matrices by simple OLS, line by line
- the covariance matrix \( \Omega \) with the OLS residuals
- the autocovariance matrices \( \Gamma \) by estimates of \( B \) matrices and the matrix \( \Omega \)
Conditions for Stationarity

The conditions for a VAR to be stationary are similar to the conditions for a univariate AR.

All the roots are to lie within the unit circle.

Moving-Average Representation of a VAR

If a VAR is stationary, the $y_t$ vector can be expressed as a sum of all of the past vector white noise shocks:

$$y_t = \mu + \sum_{i=0}^{\infty} \Psi_i \varepsilon_{t-i}$$

where $\Psi_i$ is a $(n \times n)$ matrix of coefficients, and $\Psi_0$ is the identity matrix.

In a first order AR process, in an AR probability in univariate:

Stability
Choosing the lag length

How Many Lags Needed for the VAR?

What is the most appropriate number of lags?

Trade-off:
- If \( p \) is short, model may be misspecified. Consequence: residuals will be autocorrelated over time, hence breaking OLS assumption.

- If \( p \) is long, too many degrees of freedom will be lost, i.e. every extra lag add \( n \)-squared more coefficients to estimate. Need a lot of data points.
  - Little data and long \( p \) leads to overfitting: As the VAR tries to estimate many coefficients with little data, the coefficients themselves are poorly estimated. However, the model appears to fit the in sample data very well. But out of sample forecasts are actually very bad.
How Many Lags Needed for the VAR?

As for univariate models, one can use a multi-dimensional version of the AIC and BIC criterion to find the optimal value of the number of common lags $p$:

- Schwartz Bayesian Criterion (SBC)
  \[ SBC = T \ln(\det(\Omega)) + N \ln(T) \]

- Akaike Information Criterion (AIC)
  \[ AIC = T \ln(\det(\Omega)) + 2N \]

where $N$ is the total number of parameters to estimate in all equations.

Forecasting using VARs
Forecasting using the estimated VAR

Like univariate stationary AR processes, the estimated VAR process can also be used to make forecasts.

Specifically, let

$$x_{t-1} = (y_{t-1}, y_{t-2}, \ldots, y_{t-T})$$

be a matrix containing all information available up to time $t$, before realizations of $\epsilon_t$ are known. Then:

$$E[y_t | x_{t-1}] = \hat{c} + \hat{B}_1 y_{t-1} + \hat{B}_2 y_{t-2} + \ldots + \hat{B}_p y_{t-p}$$

The forecast error is the sum of $\epsilon_t$, the unexpected innovation of $y_t$, and the coefficient estimation error

$$y_t - E[y_t | x_{t-1}] = \epsilon_t + \nu(x_{t-1})$$

If the estimator for the coefficients is consistent and estimates are based on many data observations, the coefficient estimation error tends to be small, and

$$y_t - E[y_t | x_{t-1}] \approx \epsilon_t$$
Forecasting using the estimated VAR

Iterating one period forward:

\[ E[\mathbf{y}_{t+1} | \mathbf{x}_{t-1}] = \hat{c} + \hat{\mathbf{B}}_1 E[\mathbf{y}_t | \mathbf{x}_{t-1}] + \hat{\mathbf{B}}_2 y_{t+1} + \ldots + \hat{\mathbf{B}}_p y_{t+p} + \epsilon_{t+1} \]

\[ y_{t+1} - E[\mathbf{y}_{t+1} | \mathbf{x}_{t-1}] \approx \hat{\mathbf{B}}_1 (y_t - E[\mathbf{y}_t | \mathbf{x}_{t-1}]) + \epsilon_{t+1} + \hat{\mathbf{B}}_1 \epsilon_t \]

Iterating \( j \) period forward:

\[ E[\mathbf{y}_{t+j} | \mathbf{x}_{t-1}] = \hat{c} + \hat{\mathbf{B}}_1 E[\mathbf{y}_{t+j-1} | \mathbf{x}_{t-1}] + \hat{\mathbf{B}}_2 E[\mathbf{y}_{t+j-2} | \mathbf{x}_{t-1}] + \ldots + \hat{\mathbf{B}}_p y_{t+p+j} \]

\[ y_{t+j} - E[\mathbf{y}_{t+j} | \mathbf{x}_{t-1}] \approx \epsilon_{t+j} + \hat{\mathbf{\psi}}_1 \epsilon_{t+j-1} + \hat{\mathbf{\psi}}_2 \epsilon_{t+j-2} + \ldots + \hat{\mathbf{\psi}}_j \epsilon_t \]

---

Granger Causality Test
Granger causality

X “Granger causes” Y if past values of X can help explain Y

*Granger causality is predictive causality*

Consider a F-test on the hypothesis (Granger causality):

$$H_0 : b^{(1)}_{12} = b^{(2)}_{12} = 0$$

If the $H_0$ is rejected, then we reject the null hypothesis that “x does not Granger cause y”:
Granger causality

Rejecting the Granger causality test ($\Rightarrow x \text{ “Granger causes” } y$):

- If economic theory or our knowledge, it suggests that $x$ causes $y$, use test result to confirm the theory
- If not, it simply states that $x$ improves the forecast of $y$

*Example:* an increase in orange prices today does not cause a poor harvest. But we may use our observation of this event to infer about a poor harvest.

VAR and causality: example 1

Consider a 8-order VAR with quarterly data on:

- $y =$ Peru’s GDP Q-o-Q growth (at 1994 prices)
- $x =$ Q-o-Q increase in price of crude oil (at 1994 prices)

Changes in oil prices Granger cause Peru’s growth: we can interpret this as “Peru’s GDP is affected by the real cost of oil”
VAR and causality: example 2

Consider a 8-order VAR with monthly data on:
- y = average precipitation in the US (in inches)
- x = number of tourist arrival from abroad (de-trended)

R² = 0.32 (against 0.15 if tourism is excluded): tourist arrival might help predict the weather, but it does not cause it.

The Impulse Response Function (IRF)
The impulse response function

- VAR models are often difficult to interpret: one solution is to construct the impulse responses and variance decompositions.

An impulse response function traces the responsiveness of a dependent variable in the VAR to a shock to one variables, keeping other shocks to zero.

In a VAR, one can identify a set of NxN IRFs, corresponding to the response of each variable from a shock in each variable. Usually we use 1 standard deviation shock.

- Consider for example a simple bivariate VAR(1):

\[
\begin{align*}
    y_{1t} &= \beta_{10} + \beta_{11} y_{1t-1} + \alpha_{11} y_{2t-1} + u_{1t} \\
    y_{2t} &= \beta_{20} + \beta_{21} y_{2t-1} + \alpha_{21} y_{1t-1} + u_{2t}
\end{align*}
\]

- A change in \( u_{1t} \) will immediately change \( y_{1t} \). It will change \( y_{2t} \) and also \( y_{1t} \) during the next period, etc

- We can examine how long and to what degree a shock to a given equation has on all of the variables in the system

Example: IRF for shock to real GDP

![Impulse response graph](image-url)
Constructing the impulse response function

Construct the IRF by simulating the effect on $y_{1,t}$ of a one-unit innovation $\varepsilon_{1,1}$

$$y_t = \begin{pmatrix} 0.9 & 0.2 \\ -0.3 & 0.2 \end{pmatrix} y_{t-1} + \varepsilon_t$$

$$y_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \varepsilon_t = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \varepsilon_{t+1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

How would the IRF look like if we had?

$$y_t = \begin{pmatrix} 0.9 & 0.2 \\ 0 & 0.2 \end{pmatrix} y_{t-1} + \varepsilon_t$$

Structural VAR
Reduced Form vs Structural VAR

• A reduced-form VAR expresses each variable as a linear function of its own past values and the past values of all other variables being considered and a serially uncorrelated error term. The error terms are viewed as “surprises”—movements in the variables after taking its past into account.

• A structural VAR (SVAR) uses economic theory to sort out contemporaneous links among the variables. SVARs require “identifying assumptions” that establish causal links among variables.

Structural VAR Model

- So far, we have assumed the VAR is autoregressive

- But what if the equations had a contemporaneous feedback term?
  \[ y_{1t} = \alpha_{10} + \alpha_{12} y_{2t} + \beta_{11} y_{1t-1} + \beta_{12} y_{2t-1} + e_{1t} \]
  \[ y_{2t} = \alpha_{20} + \alpha_{21} y_{1t} + \beta_{21} y_{2t-1} + \beta_{22} y_{1t-1} + e_{2t} \]

- This is the structural VAR:

  \[
  \begin{pmatrix}
  y_{1t} \\
  y_{2t}
  \end{pmatrix} =
  \begin{pmatrix}
  \alpha_{10} & \alpha_{12} \\
  \alpha_{20} & \alpha_{21}
  \end{pmatrix}
  \begin{pmatrix}
  y_{1t} \\
  y_{2t}
  \end{pmatrix}
  +
  \begin{pmatrix}
  \beta_{11} & \beta_{12} \\
  \beta_{21} & \beta_{22}
  \end{pmatrix}
  \begin{pmatrix}
  y_{1t-1} \\
  y_{2t-1}
  \end{pmatrix}
  +
  \begin{pmatrix}
  e_{1t} \\
  e_{2t}
  \end{pmatrix}
  \]

- In more general form with \( m \) variables and \( p \) lags:

  \[ y_t = \mathbf{c} + \mathbf{B}_0 y_t + \mathbf{B}_1 y_{t-1} + \mathbf{B}_2 y_{t-2} + \ldots + \mathbf{B}_p y_{t-p} + e_t \]

  This VAR can’t be estimated by OLS. Need reduced VAR. Why??
Reduced VAR Model

To obtain reduced form VAR from structural VAR, take the contemporaneous variables to the LHS:

\[
\begin{bmatrix}
1 & -\alpha_{12} \\
-\alpha_{21} & 1 
\end{bmatrix}
\begin{bmatrix}
y_{t}\ 
y_{2t}
\end{bmatrix}
= 
\begin{bmatrix}
\alpha_{t0} \\
\alpha_{20}
\end{bmatrix}
+ 
\begin{bmatrix}
\beta_{11} & \beta_{12} \\
\beta_{21} & \beta_{22}
\end{bmatrix}
\begin{bmatrix}
y_{t-1} \\
y_{2t-1}
\end{bmatrix}
+ 
\begin{bmatrix}
e_{t} \\
e_{2t}
\end{bmatrix}
\]

Multiply inverse of this term on the RHS:

\[
\begin{bmatrix}
y_{t} \\
y_{2t}
\end{bmatrix}
= 
\begin{bmatrix}
\beta_{01} \\
\beta_{20}
\end{bmatrix}
+ 
\begin{bmatrix}
\beta_{11} & \beta_{12} \\
\beta_{21} & \beta_{22}
\end{bmatrix}
\begin{bmatrix}
y_{t-1} \\
y_{2t-1}
\end{bmatrix}
+ 
\begin{bmatrix}
u_{t} \\
u_{2t}
\end{bmatrix}
\]

More generally:

\[
\left[I - B_{0}\right]y_{t} = c + B_{1}y_{t-1} + B_{2}y_{t-2} + \ldots + B_{p}y_{t-p} + \varepsilon_{t}
\]

Thus reduced VAR which can be estimated by OLS is:

\[
y_{t} = \left[I - B_{0}\right]^{-1} \left(c + B_{1}y_{t-1} + B_{2}y_{t-2} + \ldots + B_{p}y_{t-p} + \varepsilon_{t}\right)
\]

Identification in SVAR Model

- Remember that we started with a structural VAR model, and jumped into the reduced form or standard VAR for estimation purposes
- Is it possible to recover the parameters in the structural VAR from the estimated parameters in the reduced VAR?
- There are 8 parameters in the bivariate structural VAR(1) and only 7 estimated parameters in the reduced VAR(1)
- The VAR is underidentified
- If one parameter in the structural VAR is restricted the reduced VAR is exactly identified, therefore, able to recover parameters
  - Cholesky decomposition: rank the variables to that
    - First variable responds to the contemporaneous shocks of all variables
    - Second variables does not respond to shocks of 1st variables, but responds to all other shocks
    - ...
    - Last variables does not responds to any shock in the other variables
Identification in SVAR Model

\( \alpha_{21} = 0 \) implies

\[
\begin{bmatrix}
 y_{1t} \\
 y_{2t}
\end{bmatrix} = \begin{bmatrix} 1 & -\alpha_{12} \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \beta_{10} \\
 \beta_{20} \end{bmatrix} + \begin{bmatrix} 1 & -\alpha_{12} \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \beta_{11} & \beta_{12} \\
 \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} y_{1t-1} \\
 y_{2t-1} \end{bmatrix} + \begin{bmatrix} 1 & -\alpha_{12} \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} u_{1t} \\
 u_{2t} \end{bmatrix}
\]

\[
\begin{bmatrix}
 y_{1t} \\
 y_{2t}
\end{bmatrix} = \begin{bmatrix} \phi_{10} \\
 \phi_{20} \end{bmatrix} + \begin{bmatrix} \phi_{11} & \phi_{12} \\
 \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} y_{1t-1} \\
 y_{2t-1} \end{bmatrix} + \begin{bmatrix} e_{1t} \\
 e_{2t} \end{bmatrix}
\]

The parameters of the structural VAR can now be identified from the following 9 equations. Therefore, **exactly identified**!

\[
\begin{align*}
\phi_{10} &= \beta_{10} - \alpha_{12} \beta_{20} \\
\phi_{20} &= \beta_{20} \\
\phi_{11} &= \beta_{11} - \alpha_{12} \beta_{21} \\
\phi_{21} &= \beta_{21} \\
\phi_{12} &= \beta_{12} - \alpha_{12} \beta_{22} \\
\phi_{22} &= \beta_{22}
\end{align*}
\]

\[
\begin{align*}
\text{var}(e_1) &= \sigma_1^2 + \alpha_{12}^2 \sigma_2^2 \\
\text{var}(e_2) &= \sigma_2^2 \\
\text{cov}(e_1, e_2) &= -\alpha_{12} \sigma_2^2
\end{align*}
\]

Identification in SVAR Model

- Note both structural shocks can now be identified from the residuals of the standard VAR
- \( \alpha_{21} = 0 \) implies \( y_1 \) does not have a contemporaneous effect on \( y_2 \)
- This restriction manifests itself such that both \( u_1 \) and \( u_2 \) affect \( y_1 \) contemporaneously but only \( u_2 \) affects \( y_2 \) contemporaneously
- The residuals of \( e_{2t} \) are due to pure shocks to \( y_2 \)
- Decomposing the residuals of the reduced VAR in this **triangular** fashion is called the **Choleski decomposition**
- There are other methods used to identify models, like Blanchard and Quah (1989) decomposition, etc
Conclusions

- Covariance stationary VAR: properties
- Forecasting using VARs
- Granger causality test
- The impulse response function
- Structural VARs

Macroeconomic Forecasting
Thank you
Objectives

I. Learn how to evaluate a regression model
   - Residual tests
   - Stability tests
   - Coefficient tests

The Stages of Econometric Study

1. Propositions about the Economy
2. Economic Theory
3. Econometric Model
4. Data
5. Estimation of Unknown Parameters in the Model
   - Model Evaluation/Diagnostic
   - Is the Model Adequate?
      - NO
      - YES
         - Using the Model, i.e. for Forecasting and Policy Analysis
         - YES
Multiple Linear Regression Model

- Multiple linear regression model

\[ Y_i = f(x_{i1}, x_{i2}, \ldots, x_{ik}) + \varepsilon_i = \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_k x_{ik} + \varepsilon_i, \]

\( i = 1, \ldots, n \)

- \( Y \) is the dependent or explained variable
- \( x_{i1}, x_{i2}, \ldots, x_{ik} \) are the independent or explanatory variables

- The theory specifies \( f(x_{i1}, x_{i2}, \ldots, x_{ik}) \)

- \( \varepsilon \) is a random disturbance: it “disturbs” and otherwise stable relationship
- We assume that each observation in a sample is generated by the underlying process described by the function \( f \)
- The objective: estimate the parameters \( \beta \) of the model

Measuring Independent Effects

The usefulness of multiple regression models

- Multiple regression models can isolate independent effects of a set of variables on a dependent variable
- Example:

Are higher levels of education associated with higher incomes?

\[ income = \beta_1 + \beta_2 education + \varepsilon \]

But most people have higher income when they have more experience, independent of their education. \( \beta_2 \) could be overstated: if experience and education are positive correlated, the model associate all the increases to income from increases in education.

Better specification would account for experience:

\[ income = \beta_1 + \beta_2 education + \beta_3 experience + \varepsilon \]
Ordinary Least Squares

- Ordinary least squares (OLS) or linear least squares is a method for estimating the unknown parameters in a linear regression model.
- The least squares best fit is found by finding the **minimum value of the square of errors**.
  - In example below: find $a$ and $b$ such as sum of squared errors are minimum.

Best Linear Unbiased Estimator

- The Gauss-Markov Theorem states that, provided the Classical assumptions hold, the ordinary least squares (OLS) estimator $b$ is the **minimum variance estimator among all linear unbiased estimators**.
- Sometimes it is said that the OLS estimator is BLUE (**Best Linear Unbiased Estimator**).
- For this to happen, our model needs to fulfill the following conditions:
  1. Linearity
  2. Expected value of error is zero
  3. X and errors are uncorrelated
  4. Absence of serial autocorrelation
  5. Errors are homoskedastic

*If Assumptions 1, 2, 3 are satisfied, then the least squares estimator of the regression coefficients is unbiased. If assumption 4-5 are satisfied, OLS has the minimum variance among all estimators.*
Assumptions of the Classical Linear Model

1. **Linear functional form of the relationship Y and X**
   - \( Y = \alpha + \beta x + \varepsilon \) **OK**
   - \( Y = \alpha + x^2 + \varepsilon \) **NOT OK**
   - \( Y = Ax^2 \beta + \varepsilon \log Y = \log A + \beta \log x + \varepsilon \) **OK**

2. **Expected value of the disturbance is zero** at every observation, i.e. disturbances are random
   - If we expected a particular error value, then (part of) the error term would be predictable, and we could just add that to the regression model.

3. **X and \( \varepsilon \) are uncorrelated** \( (\text{Cov}(X_i, \varepsilon_i) = 0) \), i.e. \( X \) is exogenous
   - When assumption is violated, we say \( X_i \) is endogenous.
   - Why is endogeneity a problem?
     - We are attributing variation in \( Y_i \) to \( X_i \) that is really due to \( \varepsilon_i \) varying with \( X_i \). Consequently, we get a biased estimate of the coefficient on \( X \), \( \beta \), because it measures the effect of \( X \) and \( \varepsilon \) on \( Y \).

4. **No serial autocorrelation**, that is errors are statistically independent: \( E(\varepsilon_i \varepsilon_j) = 0 \) for any \( i, j \)

Assumptions of the Classical Linear Model (continued)

5. **Homoskedasticity**:
   - Variance of disturbances is constant
   - **The least squares estimator is unbiased** even if this assumption is violated.
   - But, it turns out **there are more efficient estimators (i.e. smaller variance)** than least squares if the errors are **heteroskedastic**.

Source: [http://home.ubalt.edu/ntsbarsh/business-stat/BiasVariance.gif](http://home.ubalt.edu/ntsbarsh/business-stat/BiasVariance.gif), accessed on May 07 2015
Two more assumptions

6. No perfect collinearity
With perfect collinearity, one (or more) independent variables is a perfect linear function of others. Perfect collinearity is a problem, because the least squares estimator cannot separately attribute variation in Y to the independent variables.

7. Normality of errors
In addition, the errors (disturbances) should be normally distributed, with zero mean and constant variance.

Important for hypothesis testing.

Goodness of Fit and Analysis of Variance

- The goodness of fit of a statistical model describes how well it fits a set of observations. A commonly used measure of the goodness of fit is the coefficient of determination, the $R^2$.
- Computation is based on the analysis of variance procedure that partitions the total variation in the dependent variable, denoted $SST$ (total sum of squares), into two parts: the part explained by the estimated regression equation, denoted $SSE$, and the part that remains unexplained, denoted $SSR$

$$SSR + SSE = SST$$

$$R^2 = SSR/SST$$

Note: R-squared cannot determine whether the coefficient estimates and predictions are biased, which is why you must assess the residual plots.
Hypothesis Testing

• In a regression study, hypothesis **tests are usually conducted to assess the statistical significance** of the overall relationship represented by the regression model and to test for the statistical significance of the individual parameters.

• The statistical **tests used are based on the assumptions concerning the disturbances**

• If the overall model is deemed statistically significant, conduct hypothesis **tests on the individual parameters to determine if each independent variable makes a significant contribution to the model**

Summarizing the discussion so far

• **What are we looking for** when construct and evaluate a regression?

• Before using our model for the purpose of forecasting, we would like

  1. to find “right” factors (regressors) $X$ that “explain” process $Y$ and thus have unbiased estimates of the model parameters and have minimum variance of our residual.

  2. our model to have parameters that are stable over time

     • “stable” - should not change wildly over time

     • or at least we **should be able to detect breaks** so that we can “deal” with them

  3. **to check hypotheses** about our model

• The **tools** that we have – **TESTS**:

  • **Standard** and rely on certain (calculated) **test-statistics**

  • **Idea:** compare a value of a test statistic to a critical value(s): if the test statistic **exceeds the critical value(s)**, then a tested hypothesis is **rejected**
Example

- We introduce an example of a linear regression model to see how we can evaluate models using
  1. Residual Tests
  2. Stability Tests
  3. Coefficient Tests

... in practice

Example: UK Bond Returns Forecasting Model

- Regression model for \( rB_t \) – return on a “portfolio” of long-term bonds over one month
- At time \( t \) we use available data to predict \( rB_{t+1} \)
- Factors:
  - Bond returns at lags 1 and 2: \( rB_{t-1}, rB_{t-2} \)
  - Return on the FTSE100 stock index: \( rftse_{t-1} \)
  - Return on the MSCI world stock index: \( rmsci_{t-1} \)
  - Momentum: \( mom6_{t-1} \)
  - Change in the price for oil: \( pioil_{t-1} \)
  - A dummy to account for an outlier in October 1979: \( d1979\_10 \)
Example: UK Bond Returns Forecasting Model

- **Before estimating:** have a look at the series!

- **Initial Model:**

  \[ r B_t = \beta_1 + \beta_2 r B_{t-1} + \beta_3 r B_{t-2} + \beta_4 r f s e_{t-1} + \beta_5 r m s c i_{t-1} + \ldots + \beta_8 r m c i_{t-1} + \beta_9 r f t s e_{t-1} + \beta_{10} d 1979 -10 + \epsilon_t \]

- **Full sample OLS-estimation:**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.74036914</td>
<td>0.0612319</td>
<td>12.3416785</td>
<td>0.0000</td>
</tr>
<tr>
<td>RC(1)</td>
<td>0.2450914</td>
<td>0.006138</td>
<td>4.056745</td>
<td>0.0000</td>
</tr>
<tr>
<td>RC(2)</td>
<td>-0.127012</td>
<td>0.054683</td>
<td>-2.326894</td>
<td>0.0206</td>
</tr>
<tr>
<td>RFTSE100_1</td>
<td>-0.018933</td>
<td>0.016288</td>
<td>-1.197577</td>
<td>0.2385</td>
</tr>
<tr>
<td>RMSCI_1</td>
<td>-0.030721</td>
<td>0.026816</td>
<td>-1.136444</td>
<td>0.2593</td>
</tr>
<tr>
<td>MOM6_1</td>
<td>-0.102447</td>
<td>0.068349</td>
<td>-1.498484</td>
<td>0.1668</td>
</tr>
<tr>
<td>PIOIL_1</td>
<td>0.022068</td>
<td>0.009036</td>
<td>2.454260</td>
<td>0.0134</td>
</tr>
</tbody>
</table>

  Durbin-Watson statistic: 1.979126
Residual Tests

Eyeballing the residuals

• What can you say about the mean of the residuals, serial correlation, and the variance over time?

• Get suspicious: non-constant variance - heteroskedasticity? Not efficient estimators.
Residual Tests

- To assess the validity of assumptions, we need Residual Tests
  - Residual autocorrelation. Are errors independent: $E[\epsilon_i'\epsilon_j]=0$ for any $i$, $j$?
  - Homoskedasticity. Is the error variance constant: $\text{Var}(\epsilon)=E[\epsilon'\epsilon]=\sigma^2$?
  - Normality. Are residuals normally distributed: $\hat{\epsilon}_i \approx N(0, \sigma^2)$

Autocorrelation: Q-Test and LM Tests

1. The Q-statistic looks at the correlogram of the residuals
   - Null hypothesis: data is independently distributed (correlation 0)
   - Intuition of Q test:
     - If residuals are autocorrelated, then $Q(m)$ should be “large”
     - If $Q(m)$ is “large enough” - larger than a “critical value” - then $H_0$ is rejected (so, residuals are autocorrelated indeed)
   - If residuals are autocorrelated, then $Q(m)$ should be “large”
     - If $Q(m)$ is “large enough” - larger than a “critical value” - then $H_0$ is rejected (so, residuals are autocorrelated indeed)

2. LM-tests:
   - Estimate a model with OLS and get residuals: $\hat{\epsilon}_i = y_i - \hat{\beta}_1 - \hat{\beta}_2 x_{2t} - \hat{\beta}_3 x_{3t} - ... - \hat{\beta}_k x_{kt}$
   - Regress residuals on $X$'s and lagged residuals:
     $$\hat{\epsilon}_i = \gamma_1 + \gamma_2 x_{2t} + ... + \gamma_k x_{kt} + \phi_1 \hat{\epsilon}_{t-1} + ... + \phi_m \hat{\epsilon}_{t-m} + \nu_t$$
   - Test $H_0: \phi_1=\phi_2=...=\phi_m=\nu_1=\nu_2=...=\nu_k=0$
   - Rejecting $H_0$ means: autocorrelation

66
Residual Tests: Tests for Autocorrelation

• In Eviews:

<table>
<thead>
<tr>
<th>Autocorrelation</th>
<th>Partial Correlation</th>
<th>AC</th>
<th>PAC</th>
<th>LStat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>1</td>
<td>1</td>
<td>0.000</td>
<td>0.573</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>2</td>
<td>0.02</td>
<td>0.825</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>3</td>
<td>0.06</td>
<td>0.805</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>4</td>
<td>0.10</td>
<td>0.760</td>
<td></td>
</tr>
<tr>
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<td></td>
<td>5</td>
<td>0.14</td>
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</tr>
<tr>
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<td></td>
<td>6</td>
<td>0.18</td>
<td>0.663</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>7</td>
<td>0.22</td>
<td>0.614</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>8</td>
<td>0.26</td>
<td>0.565</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>9</td>
<td>0.30</td>
<td>0.516</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>10</td>
<td>0.34</td>
<td>0.469</td>
<td></td>
</tr>
</tbody>
</table>

Question: Are errors autocorrelated?

Answer: Both tests fail to reject $H_0$ of no autocorrelation: both $Q$-stat. and $TR^2$ do not exceed critical values (we can claim it because all $p$-values >5%)

What to do when errors are serially correlated?

• Solution:
  • Add missing variables
  • Add lagged dependent (y) and independent variables (x)
Residual Tests: Tests for Homoskedasticity

• Breusch-Pagan-Godfrey Test
  • Auxiliary model:  
    \[ \hat{e}_i^2 = \gamma_1 + \gamma_2 x_{2i} + \ldots + \gamma_k x_{ki} + V_i \]
  • \( H_0: \gamma_2 = \gamma_3 = \ldots = \gamma_k = 0 \)
  • Test statistic: \( T \cdot \overline{R^2} \sim \chi^2(k-1) \)

• White's Test
  • Auxiliary model:  
    \[ \hat{e}_i^2 = \gamma_1 + \gamma_2 x_{2i} + \ldots + \gamma_k x_{ki} + \text{squares + crossprod} + V_i \]
  • \( H_0: \gamma_2 = \gamma_3 = \ldots = \gamma_k = \lambda_1 = \lambda_2 = \ldots = \lambda_k = 0 \)
  • Test statistic: \( T \cdot \overline{R^2} \sim \chi^2(2k + k(k-1)/2 - 1) \)

• ARCH LM-Test
  • Auxiliary model:  
    \[ \hat{\varepsilon}_i^2 = \omega_1 + \omega_2 \hat{\varepsilon}_{i-1}^2 + \ldots + \omega_m \hat{\varepsilon}_{i-m}^2 + V_i \]
  • \( H_0: \omega_2 = \ldots = \omega_m = 0 \)
  • Test statistic: \( T \cdot \overline{R^2} \sim \chi^2(m-1) \)

• Intuition: if there is a systematic relation of \( \hat{e}_i^2 \) and some regressors, then \( R^2 \) is going to be “large”. If, as a result, \( TR^2 \) is larger than a critical value (significant), then we reject \( H_0 \) of homoskedasticity (errors are heteroskedastic)

Residual Tests: Tests for Homoskedasticity

**In Eviews:**

**Correlograms of Residuals Squared**

<table>
<thead>
<tr>
<th>Autocorrelation</th>
<th>Partial Correlation</th>
<th>AC</th>
<th>FRC</th>
<th>Q Stat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
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<td>2.029</td>
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<td></td>
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<tr>
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<td>3.006</td>
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<tr>
<td>3</td>
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<td>0.172</td>
<td>2.039</td>
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<td></td>
</tr>
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<td>4</td>
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</tr>
<tr>
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<tr>
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<td>0.030</td>
<td>4.591</td>
<td>0.001</td>
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<td>9</td>
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<td>10</td>
<td>0.034</td>
<td>0.041</td>
<td>4.254</td>
<td>0.002</td>
<td></td>
</tr>
</tbody>
</table>

**Question?** What would you say about homoskedasticity?

Both tests reject \( H_0 \) of homoskedasticity: both Q-stat. and \( TR^2 \) exceed critical values (p-values < 5%)
What to do when errors are heteroskedastic?

- **Heteroskedasticity of errors:**
  - OLS-estimators for parameters are still unbiased/consistent...But they are not efficient:
- **Solution:**
  - Use alternative estimation method: Generalized least square (GLS) – not explicit in Eviews
  - Use heteroskedasticity and autocorrelation consistent (HAC) estimator for the error variance – possible in EViews
  - Model the variance explicitly as an ARCH/GARCH process, estimate the model using Maximum Likelihood Estimation (MLE) – see online course for more details

One solution to heteroskedasticity
ARCH Models

**ARCH models:** The conditional variance of $\varepsilon_t$ is dependent on the realized values of $\varepsilon_t$. If the realized values are large, the conditional variance in $t$ will be large as well. Illustration using an ARCH(1). Note: $\alpha$ will determine the persistency of the shocks

\[
E_{t-1}y_t = a_0 + a_1y_{t-1}
\]

and

\[
Var(y_t | y_{t-1}, y_{t-2}, \ldots) = E_{t-1}(y_t - a_0 - a_1y_{t-1})^2 = E_{t-1}(\varepsilon_t)^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2
\]

\[
E[\varepsilon_t^2 | \varepsilon_{t-1}, \varepsilon_{t-2}, \varepsilon_{t-3}, \ldots] = \alpha_0 + \alpha_1\varepsilon_{t-1}^2
\]

Residuals can come from an autoregressive process

The conditional heteroskedasticity of $\varepsilon_t$ will result in $y_t$ being heteroskedasticity itself. Thus the ARCH model is able to capture changes in volatility in the series $y_t$
ARCH Models - Generalization

A natural extension of ARCH(1) is a more general model with longer lags, ARCH(q):

\[ y_t = \mu + \sum_{j \in J} \rho_j y_{t-j} + \varepsilon_t \]

\[ \varepsilon_t \sim N(0, \sigma_t^2) \]

\[ \sigma_t^2 = \omega + \sum_{i=1}^{q} \gamma_i \varepsilon_{t-i}^2 \]

Persistence of shocks will be determined by \( \gamma_s \)

Weighted average of past shocks (news) to current volatility

Residual Tests: Tests for Normality

- Jarque-Bera-test statistic:

\[ JB = \frac{T}{6} \left( S^2 + \frac{(K-3)^2}{4} \right) \sim \chi^2(2) \]

where \( S \) is skewness and \( K \) is kurtosis.

- \( H_0 \): residuals are normal \((JB \sim 3)\)

- In Eviews:

CAREFUL!!! You found heteroskedasticity – model it… Then test for normality again

Question: Are residuals normal?

Answer: JB-test does not reject \( H_0 \) of normality: JB-stat. does not exceed critical value \((p-value > 5\%)\)
Stability Tests

- How stable is the model/model parameters over time? *In other words, is the structure of the data-generating process stable over time?*

- Testing Structural Change:
  - **Chow Breakpoint Test**: fit the same model to the pre-break data and the post-data. If the 2 models are not sufficiently different, then no structural change in the data generating process. Problem: there are structural changes that do not have a specific date.

  - **Stability Test using Recursive Coefficient**: When no specific date can be identified, use **Recursive Tests**. Example: if you have 150 obs, estimate the model using a few (say 10 obs), plot the estimated coefficients, reestimate the model using 11 obs, plot the estimated coefficients. Keep repeating, if the magnitude of a coefficient suddenly begins to change you have a break.
Stability Tests: Chow Breakpoint Test

- **Chow Breakpoint Test**
  - **Restricted Model**: \( y_t = x_t \beta + \epsilon, \forall t = 1, \ldots, T \)
  - **Unrestricted Model** (test for \( m = 1 \) breaks): \( y_t = x_t \beta_1 + \epsilon_1, \forall t = 1, \ldots, T_1 \)
    \( y_t = x_t \beta_2 + \epsilon_2, \forall t = T_1 + 1, \ldots, T \)
  - The test compares errors of a model fitted for the whole sample \([1, T]\) to the errors when models are fitted to the separate subsamples \([1, T_1], [T_1 + 1, T_2], \ldots, [T_m + 1, T]\).
  - \( H_0: \) there is no break
  - **F-statistic**: \( F = \left( \hat{\epsilon}_R \cdot \hat{\epsilon}_R - \hat{\epsilon}_U \cdot \hat{\epsilon}_U \right) / \left( \hat{\epsilon}_U \cdot \hat{\epsilon}_U / (T - (m + 1)k) \right) \approx F(mk, T - (m + 1)k) \)
    \( \hat{\epsilon}_U \cdot \hat{\epsilon}_U = \sum_{i=1}^{m+1} \hat{\epsilon}_{UI} \cdot \hat{\epsilon}_{UI} \), where \( i \) corresponds to a subsample
  - Intuition: if the \( U \)-model (that accounts for a break) produces a sum of squared errors that is substantially smaller (so that there is a break indeed) than that for the \( R \)-model, then \( F \)-stat is going to be “large”...

Stability Tests: Chow Forecasting Test

- **Chow Forecast Test**
  - **Full-Sample Model**: \( y_t = x_t \beta + \epsilon, \forall t = 1, \ldots, T \)
  - **Sub-Sample Model**: \( y_t = x_t \beta_1 + \epsilon_1, \forall t = 1, \ldots, T_1, \quad T_1 < T \)
  - The test compares in-sample residuals of the 2 models
  - \( H_0: \) there is no break
  - **F-statistic**: \( F = \left( \hat{\epsilon}_{FS} \cdot \hat{\epsilon}_{FS} - \hat{\epsilon}_{SS} \cdot \hat{\epsilon}_{SS} \right) / \left( \hat{\epsilon}_{SS} \cdot \hat{\epsilon}_{SS} / (T_1 - k) \right) \approx F(T - T_1 + 1, T_1 - k) \)
    where \( \hat{\epsilon}_{FS} = y_t - x_t \hat{\beta} \) for \( t = 1, \ldots, T \) and \( \hat{\epsilon}_{SS} = y_t - x_t \hat{\beta}_1 \) for \( t = 1, \ldots, T_1 \)
  - Intuition: if there is no break then the fit in both the full-sample and the sub-sample should be of the same degree of accuracy. If there is a break, then the fit of the full-sample model should deteriorate such that \( \hat{\epsilon}_{FS} \cdot \hat{\epsilon}_{FS} \) becomes “large”, which induces the \( F \)-statistic to become “large”...
Stability Tests: Chow Tests

- Eviews illustration of the CBT and CFT: is there a break in December 1995?

**Chow Breakpoint Test**

<table>
<thead>
<tr>
<th>Null Hypothesis: No breaks at specified breakpoints</th>
<th>Varying regressor: All equation variables</th>
</tr>
</thead>
</table>

- **Test Equation:** Specification R0101: R0101 (-1) R0101 (1) - R0101 (2) PIOLS_1
- **Test predictions for observations from 1995M12 to 2000M12**

<table>
<thead>
<tr>
<th>F-statistic</th>
<th>Prob. (F(4, 33))</th>
<th>Prob. (Chi-Square)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.70463</td>
<td>0.4147</td>
<td>0.1394</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Log likelihood ratio</th>
<th>Prob. (Chi-Square)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03432</td>
<td>0.6184</td>
</tr>
</tbody>
</table>

**F-test summary:**

<table>
<thead>
<tr>
<th>Value</th>
<th>df</th>
<th>Prob. (Chi-Square)</th>
</tr>
</thead>
<tbody>
<tr>
<td>141.161</td>
<td>133</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

**Note:** to run the stability tests the model should be re-estimated using Weighted OLS with weights brought by the estimate of errors std. deviation to adjust for the heteroskedasticity (more on this in the workshop).

- Question: Can you reject the hypothesis of a structural break in 1995M12?
- Answer: Both tests do not reject $H_0$ of no structural break in 1995M12.

Stability Tests: Recursive Coefficients

- Recursive Coefficients
  - The regression model is recursively estimated on $m$ subsamples $[1, T_1], [1, T_1+1], ..., [1, T]$ to produce a sequence of $\hat{\beta}_m$
    
    $$y_m = x'_m \hat{\beta}_m + \epsilon_m$$

  - If it is found that the value of (some) estimated coefficients change wildly over time, then it indicates breaks

- Eviews example:
The CUSUM tests are based on the cumulative sum of the recursive residuals.

The test finds parameter instability if the cumulative sum goes outside the area between the two critical lines.

The CUSUM test rejects stability at 5% significance level if the test statistic $W_t$ crosses limits given by $\pm 0.948\sqrt{T-k} + 2(t-k)\sqrt{T-k}$.

The CUSUM of Squares test rejects stability if the test statistic $S_t$ crosses limits given by $\pm c + (t-k)/(T-k)$, where $c$ depends on the chosen significance level.

Eviews example:
Hypothesis Testing

Hypothesis Testing: Coefficient Tests

- We use these tests to evaluate hypotheses about parameter values:
  - should a variable be excluded from a regression – a corresponding parameter value is (statistically) zero?
  - could a certain parameter(s) be restricted to a particular economic-theory-consistent value(s)?

- Coefficient Tests we are looking at here:
  - Test for significance of a particular coefficient: t-test
  - Test of linear restrictions for coefficients: Wald test (restricted comes from the idea of a restricted model, fewer variables)
  - Test for omitted/redundant variables: Log-likelihood ratio test
Significance of a particular coefficient

- Simple univariate model:
  \[ y_t = \beta x_t + \varepsilon_t \]

- We want to know if factor \( x_t \) is important (statistically significant) for explaining \( y_t \): If \( \beta = 0 \) then \( x_t \) is not important

- \( t \)-statistic for a \( t \)-test:
  \[ t_{\hat{\beta}} = \frac{\hat{\beta} - \beta}{\text{s.e.}(\hat{\beta})} \]

- Logic: if \( \hat{\beta} \) is far from 0, then \( t_{\hat{\beta}} \) is going to be either quite large (if \( \beta > 0 \)) or quite small (if \( \beta < 0 \))

- Also, for \( t_{\hat{\beta}} \) to be valid it is rather important that \( \hat{\beta} \) and \( \text{s.e.}(\hat{\beta}) \) are “correct” (e.g. have good properties – consistent estimates of true values)

- Properties of model errors are key for desirable properties of parameters’ estimators and for correct implementation of model diagnostics!!

Coefficient Tests: Wald Test

- To test whether two or more coefficients within a model are equal, if we can exclude variables from our model

- Our model in a post-residual-tests representation:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>p-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>mom6t</td>
<td>0.176277</td>
<td>0.069742</td>
<td>0.055142</td>
<td>0.2880</td>
</tr>
<tr>
<td>rftset</td>
<td>-0.150681</td>
<td>0.069742</td>
<td>-0.055142</td>
<td></td>
</tr>
<tr>
<td>rmscit</td>
<td>0.010531</td>
<td>0.069742</td>
<td>0.055142</td>
<td>0.7634</td>
</tr>
<tr>
<td>mom6t_m</td>
<td>0.176277</td>
<td>0.069742</td>
<td>0.055142</td>
<td>0.2880</td>
</tr>
<tr>
<td>rftset_m</td>
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<td>0.069742</td>
<td>-0.055142</td>
<td></td>
</tr>
<tr>
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<td>0.010531</td>
<td>0.069742</td>
<td>0.055142</td>
<td>0.7634</td>
</tr>
<tr>
<td>mom6t_r</td>
<td>0.176277</td>
<td>0.069742</td>
<td>0.055142</td>
<td>0.2880</td>
</tr>
<tr>
<td>rftset_r</td>
<td>-0.150681</td>
<td>0.069742</td>
<td>-0.055142</td>
<td></td>
</tr>
<tr>
<td>rmscit_r</td>
<td>0.010531</td>
<td>0.069742</td>
<td>0.055142</td>
<td>0.7634</td>
</tr>
<tr>
<td>mom6t_c</td>
<td>0.176277</td>
<td>0.069742</td>
<td>0.055142</td>
<td>0.2880</td>
</tr>
<tr>
<td>rftset_c</td>
<td>-0.150681</td>
<td>0.069742</td>
<td>-0.055142</td>
<td></td>
</tr>
<tr>
<td>rmscit_c</td>
<td>0.010531</td>
<td>0.069742</td>
<td>0.055142</td>
<td>0.7634</td>
</tr>
</tbody>
</table>

- We test if coefficients for mom6t, rftset, rmscit, and are jointly statistically insignificant. IN EVIEWs, we need to put the following restriction: c(4)=c(5)=c(6)=0 in EviewS
Coefficient Tests: Wald Test

- Eviews illustration of Wald test for the rB regression

- Question: Would you be able to reject the null hypothesis?
- Answer: We do not reject the null of coefficients being 0 jointly

Coefficient Tests: Log-Likelihood Ratio Test

- Omitted Variables Test: we start from a Restricted model
- Redundant Variables Test: we start from an Unrestricted model
- The $H_0$ is that redundant/omitted variables do not belong to a U-model
- Test statistic in either case – the Likelihood Ratio:

$$LR = -2(\ln L_R - \ln L_U) \xrightarrow{A} \chi^2(r),$$

where $r$ is a number of omitted variables

- Intuition:
  - Omitted variable test: if adding an extra variable (going from R to U) improves the log-likelihood substantially (makes LR large), then this variable should be in the model – we reject the null
  - Redundant variable test: if dropping a variable (going from U to R) decreases the log-likelihood substantially (makes LR smaller), then this variable should not be excluded from the model – we reject the null
Coefficient Tests: LLR-test

- Eviews illustration of the Redundant variable LR-test for the rB regression

Model evaluation is an important part of modeling business.

Should start with testing of residuals to see if all assumptions are valid.

If they are – congratulations!!! – you can proceed with the OLS.

If they are not – more work is needed:
  - reformulation
  - alternative estimation methods, etc.

Further model evaluation: coefficient hypothesis testing and stability testing.
Thank you
Workshop on Macro-Fiscal Forecasting
Arusha, August 8 – 18, 2016

L9. The Effective Tax Rate Approach

Phyllis Resnick (Consultant)

Outline

I. Introduction: revenues, baseline projections, and types of forecasts
II. The Effective Tax Rate approach: definition and use
III. Making assumptions about the Effective Tax Rate
Part 1. Introduction: revenues, baseline projections, and types of forecasts

Revenues and expenditures

Revenues
Non repayable receipts (i.e. receipts which do not give rise to an obligation of repayment):

Expenditures
Payments that do not (by themselves) decrease the stock of liabilities (amortization of debt is not an expenditure)
Financing

Non repayable receipts: taxes, profits, and grants

Expenditures

FINANCING: Domestic borrowing
- Central bank (monetization)
- Bank financing
- Non-bank financing
Foreign borrowing
Privatization receipts

OVERALL BALANCE =

Objective of Forecasting

Inform policymakers of fiscal effects of policy changes and/or of economic developments

“BASELINE SCENARIO”
- Assume current policies or confirmed policy changes
- Use macroeconomic projections at current policies

“POLICY SCENARIO”
- Simulate effect of changes to policies
- Use macroeconomic projections consistent with assumed policy change
Forecasts Serve Multiple Purposes

Inform policymakers of fiscal effects of policy changes and/or of economic developments

Multiple Purposes and Time Horizons
- Budgeting – Generally 1 or 2 year horizon
- Revenue Monitoring - < 1 year horizon
- Planning – Horizon can be 10 years or longer

Focus here is on shorter time horizons

Definition of Revenues

Non repayable receipts except grants and transfers (they do not give rise to an obligation of repayment)

<table>
<thead>
<tr>
<th>Total Revenues and Grants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax revenues</td>
</tr>
<tr>
<td>Direct taxes</td>
</tr>
<tr>
<td>Taxes on income</td>
</tr>
<tr>
<td>Taxes on wealth</td>
</tr>
<tr>
<td>Indirect taxes</td>
</tr>
<tr>
<td>Taxes on goods and services (VAT, sales tax, turnover)</td>
</tr>
<tr>
<td>Taxes on imports</td>
</tr>
<tr>
<td>Other tax revenues</td>
</tr>
<tr>
<td>Nontax revenue</td>
</tr>
<tr>
<td>License, fees, etc.</td>
</tr>
<tr>
<td>Central Bank profits</td>
</tr>
<tr>
<td>Grants and transfers</td>
</tr>
</tbody>
</table>
Unconditional forecasting methods

Use only past dynamic of the variable to project the future dynamic (example, ARIMA models)

\[ y_t = a + b_1 y_{t-1} + b_2 y_{t-2} + \ldots + b_p y_{t-p} + \epsilon_t \]

Conditional forecasting methods

Use the past dynamic of the variable and its relation with other variables to project the future dynamic

- Econometric models (distributed lag models)
- Microeconomic models (using data from actual tax returns to simulate the effects of changes to rates or the effect of economic developments on revenues)
- Effective tax rate approach
- Elasticity approach
Which method to use?

There is no “best” method

Which method to use depends on:
- The question
- Data availability
- The resources available (time, software, capacity)

Use more than one method, prefer simplicity first, and use best judgment

Part 2. The Effective Tax Rate approach: definition and use
# How tax revenues are originated

Tax revenue = Statutory rate * Tax base

**TAX BASE:** the event or condition that gives rise to taxation and is defined in the law.
- Taxable event: Receipt of wages, sale of goods
- Taxable condition: Ownership of a house

**STATUTORY RATE:** the percent of tax base (for taxes *ad-valorem*) or the fixed amount per unit (per unit taxes) that must be paid

---

# Forecasting based on statutory tax rates

...requires a lot of information: tax rates and brackets, income distribution, deductions

<table>
<thead>
<tr>
<th>Income brackets</th>
<th>&lt; 60</th>
<th>60 - 100</th>
<th>&gt;100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statutory tax rate</td>
<td>15%</td>
<td>30%</td>
<td>50%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Income 2015</th>
<th>Person 1</th>
<th>Person 2</th>
<th>Person 3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taxable income</td>
<td>20</td>
<td>40</td>
<td>120</td>
<td>180</td>
</tr>
<tr>
<td>Applicable rate</td>
<td>15%</td>
<td>15%</td>
<td>50%</td>
<td>---</td>
</tr>
<tr>
<td>Personal tax</td>
<td>3</td>
<td>6</td>
<td>60</td>
<td>69</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Income 2016</th>
<th>Person 1</th>
<th>Person 2</th>
<th>Person 3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taxable income</td>
<td>20</td>
<td>70</td>
<td>90</td>
<td>180</td>
</tr>
<tr>
<td>Applicable rate</td>
<td>15%</td>
<td>30%</td>
<td>30%</td>
<td>---</td>
</tr>
<tr>
<td>Personal tax</td>
<td>3</td>
<td>21</td>
<td>30</td>
<td>54</td>
</tr>
</tbody>
</table>
The Effective Tax Rate

Effective Tax Rate = Tax revenue / Proxy tax base

PROXY TAX BASE: Economic variable that is closely related to (but it is NOT) the actual tax base

EFFECTIVE TAX RATE: the ratio between the tax revenue and the proxy tax base

Note: ETR can be defined for total or a specific tax revenue

Rationale for the ETR approach

Most commonly used revenue forecasting method:

• Data are available

• It is simple and tractable
  - No need to project numerous actual tax bases
  - No need to collect information on statutory tax rates and exemptions for various tax items

• Can be applied for meaningful tax aggregates
  - Actual tax base is defined at very detailed level while proxy can be defined for broad aggregates
Forecasting using the ETR

Tax revenue = Assumed ETR * Forecast of proxy tax base

Main steps:
- Select the proxy tax base
- Forecast the proxy tax base
- Forecast the ETR
- Obtain the revenue forecast multiplying the forecast of the proxy tax base by the assumed ETR

Example of ETR-based forecast

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Imports</td>
<td>16000</td>
<td>19000</td>
<td>25000</td>
<td>30000</td>
<td>35000</td>
<td>40000</td>
</tr>
<tr>
<td>ETR</td>
<td>4.8</td>
<td>5</td>
<td>4.9</td>
<td>5.6</td>
<td>5.8</td>
<td>6</td>
</tr>
<tr>
<td>Custom duties</td>
<td>768</td>
<td>950</td>
<td>1225</td>
<td>1680</td>
<td>2030</td>
<td>2400</td>
</tr>
</tbody>
</table>

6% : Trend ETR
It is important to ask...
- What do we assume?
- Is the assumption reasonable?
Part 3. Making assumptions about the Effective Tax Rate

Choosing the proxy tax base

**PROXY TAX BASE:** Economic variable that is closely related to (but it is NOT) the actual tax base

A good proxy tax base must be:
- Highly correlated with the actual tax base (either because of legal or economic reasons)
- Justifiable (the correlation can be explained either because of legal or economic reasons)
- Easy to forecast and link to main macroeconomic developments and macroeconomic policies
Example of a good/bad proxy (I)

The case of Fiji: tourism is one of the most important sectors of the economy, and the sector where most corporations (small/big) operate

Example of a good/bad proxy (I)

Fiji: data on private consumption or household income are not good. However, registration of new cars is available...Why would new cars be a proxy base for PIT?
## Proxy tax bases

<table>
<thead>
<tr>
<th>Useful proxies</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual income tax</td>
<td>Personal income, nominal GDP</td>
</tr>
<tr>
<td>Corporate income tax</td>
<td>Corporate profits, nominal GDP</td>
</tr>
<tr>
<td>VAT, Excise taxes, sale tax</td>
<td>Nominal private consumption, nominal GDP (imports or exports of goods and services)</td>
</tr>
<tr>
<td>Import duties</td>
<td>Imports</td>
</tr>
<tr>
<td>Export duties</td>
<td>Exports</td>
</tr>
<tr>
<td>Excises</td>
<td>Consumption of selected item, real private consumption expenditure, real GDP</td>
</tr>
<tr>
<td>Other taxes</td>
<td>Nominal GDP; production of natural resources</td>
</tr>
<tr>
<td>Non-tax revenues</td>
<td>Nominal GDP</td>
</tr>
</tbody>
</table>

## How to forecast the ETR (I)

1. Compute historical ETR
2. Make sense of past ETR:
   - Is it stable or unstable?
   - Does it follow a trend?
   - Have there been breaks?
   - What can explain past ETR?
3. Project...
Projecting ETR

- Trend projection: $\text{ETR}_{\text{year}} = \alpha + \beta \times \text{(year)}$

- Judgments
  - Period?
  - Ever-growing?
  - Ever-decreasing?
  - Constant?

- Other methods

What could affect the ETR? (I)

**THE STATUTORY TAX RATE**, including exemptions, allowances, and other legal aspects defining how much should be paid

Statutory tax rate differs from the ETR reflecting...
- Proxy vs. actual tax base and aggregation
- Exemptions or deductions
- Illegal tax-free transaction
- Compliance rate

If the statutory rate increases or allowances decrease then the ETR would increase
What could affect the ETR? (II)

**COMPLIANCE RATE**, gap between the actual revenue and the potential revenue

The compliance rate reflects:
- Effectiveness of tax administration
- Disincentives to evasion
- Incentives to comply (including simplicity of the tax)

If compliance improves, then the ETR increases

What could affect the ETR? (III)

**BASE SHIFT**, changes in the composition of the tax base

Base shifts generally reflect:
- Structural changes in the economy (example, greater importance of a sector that is taxed more heavily)
- Shocks

If the tax base shifts toward categories with higher statutory tax rate the ETR would increase
Tax base structure and ETR

Example: Income distribution

<table>
<thead>
<tr>
<th>Income brackets</th>
<th>&lt;60</th>
<th>60 - 100</th>
<th>&gt;100</th>
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<td>20</td>
<td>40</td>
<td>120</td>
<td>180</td>
<td>---</td>
</tr>
<tr>
<td>Applicable rate</td>
<td>15%</td>
<td>15%</td>
<td>50%</td>
<td>---</td>
<td>38.3</td>
</tr>
<tr>
<td>Personal tax</td>
<td>3</td>
<td>6</td>
<td>60</td>
<td>69</td>
<td>---</td>
</tr>
</tbody>
</table>

<table>
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<td>Applicable rate</td>
<td>15%</td>
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<td>30.0</td>
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<td>Personal tax</td>
<td>3</td>
<td>21</td>
<td>30</td>
<td>54</td>
<td>---</td>
</tr>
</tbody>
</table>

Implication of assumptions about ETR

Constant ETR assumes
- Unchanged statutory tax rate (policy)
- Unchanged compliance rate (administration)
- Unchanged tax base structure (economic)

Keep in mind that ETR could increase reflecting...
- Statutory rate increase
- Administrative improvements
- Base shift from low- to high- tax category
## Pros and cons of ETR approach

<table>
<thead>
<tr>
<th>PROS</th>
<th>CONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Easy to use</td>
<td>• Need to use a lot of judgment (on effect of reforms, economic</td>
</tr>
<tr>
<td>• Typically used for aggregates</td>
<td>development, etc.)</td>
</tr>
<tr>
<td>• Limited data needs</td>
<td>• Assumptions can be debatable (easy to hide unrealistic assumptions)</td>
</tr>
<tr>
<td>• No need to keep track of statutory tax rates and exemptions for</td>
<td></td>
</tr>
<tr>
<td>detailed tax items</td>
<td></td>
</tr>
</tbody>
</table>

## Summary
Conclusion

• No best forecasting methods: use more than one, but prefer simple methods first

• Effective Tax Rate approach: assume ETR and multiply by forecast of the (proxy) tax base

• To make assumptions about ETR study past ETR and explain changes (breaks) looking at changes in statutory rates, administrative changes, inflation, changes in the composition of the tax base

Thank you
Workshop on Macro-Fiscal Forecasting
Arusha, August 8 – 18, 2016

L10. The Elasticity Approach
Phyllis Resnick (Consultant)

Outline

I. The elasticity approach: definition and use
II. Interpreting elasticity and buoyancy
III. Computing buoyancy and estimating the elasticity
Part 1. The elasticity approach: definitions and use

What are we Measuring?

• Both buoyancy and elasticity are measures of RESPONSIVENESS
  • Of tax revenue to its underlying tax base
• Why two measures?
  • Buoyancy can be directly calculated from the data
    • Buoyancies do not correct for exogenous (discretionary) events such as policy changes
  • Elasticity can only be estimated from the data
    • Elasticities correct for exogenous (discretionary) events
Buoyancy

\[ \text{Buoyancy}_t = \frac{\% \text{ change in tax revenue}_t}{\% \text{ change in proxy tax base}_t} \]

**Interpretation:** the observed percent change in revenue at time \( t \) compared to the observed percent change in the proxy tax base at time \( t \), all included.

- Buoyancy reflects **all changes** in the tax system, including the tax rates and brackets, the definition of the base, variations in enforcement/compliance, or other specific shocks.

Elasticity

\[ \text{Elasticity} = \frac{\% \text{ change in tax revenue}}{\% \text{ change in proxy tax base}} \]

**Interpretation:** the percent change in tax revenue caused by a 1 percent change in the proxy tax base, keeping all else equal

- Elasticity assumes **no changes in policies**, that is changes in tax revenues mainly reflect changes in the tax base and not the effect of changes, for example, in statutory rates or compliance.
Forecasting using buoyancy

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Import duties</td>
<td>97.0</td>
<td>100.8</td>
<td>103.8</td>
<td>114.2</td>
<td>126.0</td>
<td>140.7</td>
</tr>
<tr>
<td>Imports</td>
<td>1,000.0</td>
<td>1,050.0</td>
<td>1,070.0</td>
<td>1,120.0</td>
<td>1,200.0</td>
<td>1,300.0</td>
</tr>
<tr>
<td>% Change in Import duties</td>
<td>3.1</td>
<td>3.9</td>
<td>3.0</td>
<td>10.1</td>
<td>10.3</td>
<td>11.7</td>
</tr>
<tr>
<td>% Change in Imports</td>
<td>3.1</td>
<td>5.0</td>
<td>1.9</td>
<td>4.7</td>
<td>7.1</td>
<td>8.3</td>
</tr>
<tr>
<td>Buoyancy</td>
<td>1.0</td>
<td>0.8</td>
<td>1.6</td>
<td>2.2</td>
<td>1.4</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Assume a buoyancy of 1.4 for 2016:

Projected % change in import duties: 8.3% * 1.4 = 11.7

Projected import duties: 126 * (1 + 11.7%) = 140.7

Rationale for the buoyancy approach

Very commonly used revenue forecasting method:

- Simple and transparent.
- Can be computed even with limited data.
- Very good to estimate effects of shocks to the proxy tax base.
- Can be applied to any level of tax (aggregate, or itemized).
- Can be used to compute “what if” analysis, especially to compute cyclically adjusted revenues.
## Part 2. Interpreting elasticity and buoyancy

### Interpretation

<table>
<thead>
<tr>
<th>If...</th>
<th>the tax is...</th>
<th>which means that...</th>
</tr>
</thead>
<tbody>
<tr>
<td>e &gt; 1 (b &gt; 1)</td>
<td>Elastic (Buoyant)</td>
<td>the tax revenue increases more than proportionately to a rise in the proxy tax base</td>
</tr>
<tr>
<td>e = 1 (b = 1)</td>
<td>Unitary Elastic (Buoyant)</td>
<td>the tax revenue increases proportionately to a rise in the proxy tax base</td>
</tr>
<tr>
<td>e &lt; 1 (b &lt; 1)</td>
<td>Inelastic (not buoyant)</td>
<td>the tax revenue increases less than proportionately to a rise in the proxy tax base</td>
</tr>
</tbody>
</table>
### Elasticity, buoyancy, and the ETR (I)

Let T mean “tax revenue” and PB mean “proxy tax base”

\[ e = \frac{T_t - T_{t-1}}{PB_t - PB_{t-1}} \]

\[ e = \frac{T_t}{PB_t} - 1 \]

- If \( e = 1 \) \[ \frac{PB_t}{PB_{t-1}} - 1 = \frac{T_t}{T_{t-1}} - 1 \]

- If \( e > 1 \) \[ \frac{T_t}{T_{t-1}} - 1 > \frac{PB_t}{PB_{t-1}} - 1 \]

- If \( e > 1 \) \[ ... < ... \]

### Elasticity, buoyancy, and the ETR (II)

- If \( e > 1 \) \( \implies \) ETR increases over time
- If \( e = 1 \) \( \implies \) ETR remains constant over time
- If \( e < 1 \) \( \implies \) ETR decreases over time

---

**ETR under different elasticities**

- Elasticity 1.2
- Elasticity 1
- Elasticity 0.8

---

In percent of proxy tax base

- 2016
- 2017
- 2018
- 2019
- 2020
- 2021
- 2022
- 2023
- 2024
- 2025
Why can taxes be elastic or inelastic? (I)

Important ingredients:
- different rates applied to different values (subsets) of the tax base
- shifts (or composition changes) in the base
- delays in adjusting the rates
- the tax is a fixed amount
- collection lags in a context of high inflation
- delay in adjusting tax brackets with moderate inflation

Why can taxes be elastic or inelastic? (II)

The effect of progressivity and base shift

<table>
<thead>
<tr>
<th>Income brackets</th>
<th>&lt; 60</th>
<th>60 - 100</th>
<th>&gt;100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statutory tax rate</td>
<td>15%</td>
<td>30%</td>
<td>50%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>From tax code</th>
<th>Person 1</th>
<th>Person 2</th>
<th>Person 3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taxable income</td>
<td>50</td>
<td>90</td>
<td>120</td>
<td>260</td>
</tr>
<tr>
<td>Applicable rate</td>
<td>15%</td>
<td>30%</td>
<td>50%</td>
<td>---</td>
</tr>
<tr>
<td>Personal tax</td>
<td>7.5</td>
<td>27</td>
<td>60</td>
<td>94.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Income 2016</th>
<th>Person 1</th>
<th>Person 2</th>
<th>Person 3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taxable income</td>
<td>75</td>
<td>135</td>
<td>180</td>
<td>390</td>
</tr>
<tr>
<td>Applicable rate</td>
<td>30%</td>
<td>50%</td>
<td>50%</td>
<td>---</td>
</tr>
<tr>
<td>Personal tax</td>
<td>22.5</td>
<td>67.5</td>
<td>90</td>
<td>180</td>
</tr>
</tbody>
</table>

Percent change in tax: 90.5
Percent change in tax base: 50.0
Buoyancy: 1.8
Why can taxes be elastic or inelastic? (III)

The effect of shifts in consumption patterns

<table>
<thead>
<tr>
<th>From tax code</th>
<th>Type of good</th>
<th>Services</th>
<th>Goods</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statutory tax rate</td>
<td>Services</td>
<td>0%</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Goods</td>
<td>20%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2015</th>
<th>Services</th>
<th>Goods</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>250</td>
<td>250</td>
<td>500</td>
</tr>
<tr>
<td>Applicable rate</td>
<td>0%</td>
<td>20%</td>
<td>---</td>
</tr>
<tr>
<td>Consumption tax</td>
<td>0</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2016</th>
<th>Services</th>
<th>Goods</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taxable income</td>
<td>350</td>
<td>300</td>
<td>650</td>
</tr>
<tr>
<td>Applicable rate</td>
<td>0%</td>
<td>20%</td>
<td>---</td>
</tr>
<tr>
<td>Personal tax</td>
<td>0</td>
<td>60</td>
<td>60</td>
</tr>
</tbody>
</table>

Percent change in tax: 20.0
Percent change in tax base: 30.0
Buoyancy: 0.7

Short and long run elasticity (I)

A tax can have an elasticity different from 1 in the short run; what would happen in the long run?

- taxes collected would either go to zero (e < 1), or...
- taxes would exceed the tax base (e > 1)

Over the long run, elasticity should converge to 1.

- Lag in indexation and distortions would be corrected
- Inflation would return to normal
- Base changes would stabilize (or tax code changed)
- Social pressure against high ETR
Part 3. Computing buoyancy and estimating the elasticity

Computing buoyancy

Buoyancy can be computed year by year using formula

$$buoyancy_t = \frac{\% \ change \ in \ tax \ revenue, \ (T)}{\% \ change \ in \ proxy \ tax \ base, \ (PB)}$$

Note how the buoyancy cycles with the economy. Should give caution in interpreting single year buoyancies, particularly in periods of economic volatility.
Estimating elasticity

Elasticity is a measure of the response of taxes to the base excluding the effects of policy changes; it should be estimated with COINTEGRATION ANALYSIS (an econometric technique) controlling for factors that capture policy changes.

Pros and cons of elasticity approach

<table>
<thead>
<tr>
<th>PROS</th>
<th>CONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Can be used for aggregates or single items</td>
<td>• Need to estimate the elasticity</td>
</tr>
<tr>
<td>• Simple to use</td>
<td>• Need to justify elasticity assumptions and differences between short / long run</td>
</tr>
<tr>
<td>• Best for scenario analysis with no policy change (analysis of shocks or baseline)</td>
<td>• Difficult to include effects of policy changes</td>
</tr>
</tbody>
</table>

• Note: for policy changes one needs micro models
Estimating elasticity: step 0

**STEP 0:** take logT (call it \( t \)) and logPB (call it \( pb \))

Consider what elasticity really means for how to model taxes:

\[
T_t = \alpha (PB_t)^\varepsilon
\]

Take log:

\[
\log T_t = \log \alpha + \varepsilon \log (PB_t)
\]

Take difference; if \( \alpha \) is constant:

\[
\varepsilon = \frac{\log T_t - \log T_{t-1}}{\log (PB_t) - \log (PB_{t-1})}
\]

Simple estimate of elasticity

**This generally works:** estimate

**STEP 1:** Estimate

\[
\log T_t = \log \alpha + \varepsilon \log (PB_t)
\]

by OLS

**Strong assumption:** all policy changes are somewhat random or they cancel each other out over time
An Often Used Approach to Elasticity Analysis

- Estimate \( \log T_t = \log \alpha + \varepsilon \log(PB_t) \)
- Use \( \varepsilon \) as the elasticity
  - If \( \varepsilon \) is significantly different from 1, assume that in the long run \( \varepsilon \) will approach 1
- Why?
  - Remember relationship with ETR

Problems with this Approach

- Economic
  - Assertion that elasticity is 1 in the long run may not always be true
    - Behavioral changes over time may result in failure to approach 1
    - Under some circumstances, the elasticity may degrade and taxes may lose productivity
- Econometric
  - Estimating the simple log log elasticity equation in OLS will likely violate requirements for this estimation
Perhaps a more appropriate method: Cointegration Analysis

If y and x are cointegrated then:

• The long run relationship is described by

\[ y_t = c_2 + B_1 x_t \]

How y responds to x over the long run. When is \( B_1 \) an elasticity?

• In the short run, y adjusts to:
  – Past changes in x (and of y)
  – Past deviation from long run equilibrium

\[ \Delta y_t = c_1 + \phi [y_{t-1} - c_2 - \varepsilon x_{t-1}] + \beta_1 \Delta x_{t-1} + \rho_1 \Delta y_{t-1} \]

How y responds to deviation from equilibrium
How y responds to x in the short run

Simple estimate of short run elasticity

Estimate the following:

\[ \Delta t_t = c_1 + \phi [t_{t-1} - c_2 - \varepsilon p b_{t-1}] + \beta_1 \Delta p b_{t-1} + \rho_1 \Delta t_{t-1} \]

Note:

• The short run response of the tax to the base is \( \beta_1 \)

• the proper estimation technique is Fully-Modified Least Squares on the equation above

• Unless we control for other factors which could affect taxes, we are assuming that policy changes cancel out, which could be a strong assumption. Better to control for other factors (tax rate, inflation, base shifts)
From estimates to forecasts

General rule to make things simple:
Use $\varepsilon$ for the short run and 1 for the long run, if not justify why

Note: if using $\beta 1$ the full model should be used to forecast the tax, including the effect of the lag change of the tax and the adjustment to deviation from equilibrium. This implies that we must trust the model and its estimates!

Implication of assumptions about elasticity

Remember that
- Elasticity > 1 implies that the ETR increases
- Elasticity = 1 implies that the ETR is constant
- Elasticity < 1 implies that the ETR decreases

Keep in mind that ETR could increase reflecting...
- Statutory rate increase
- Administrative improvements
- Base shift from low- to high- tax category
Summary

Conclusion

• No best forecasting methods: use both elasticity and ETR and see what the assumption about one implies for the other

• Elasticity should be estimated, but this can be safely done with simple OLS

• Remember the simple math of elasticity and ETR: if the elasticity is >1 then ETR increases, if it is =1 the ETR remains constant, and if the elasticity <1 the ETR declines.
Appendix

Taking log to obtain elasticity

Consider

\[\varepsilon = \frac{\log T_t - \log T_{t-1}}{\log (PB_t) - \log (PB_{t-1})}\]

Remember, the difference of a log is the log of the ratio

\[\varepsilon = \frac{\log \left( \frac{T_t}{T_{t-1}} \right)}{\log \left( \frac{PB_t}{PB_{t-1}} \right)} \quad \text{or} \quad \varepsilon = \frac{\log \left( \frac{T_t - T_{t-1} + 1}{T_t} \right)}{\log \left( \frac{PB_t - PB_{t-1} + 1}{PB_t} \right)}\]

Remember, log of 1 plus a small \(x\) is about \(x\)

\[\varepsilon = \frac{T_t - T_{t-1}}{T_{t-1}} \cdot \frac{PB_{t-1}}{PB_t - PB_{t-1}}\]
Thank you
Outline of the lecture

1. An Introduction to Fiscal Accounts
2. Forecasting Interest Payments, Borrowing and Debt
3. Debt Dynamics
4. Ways of Budget Financing
5. Macroeconomic implications of budget financing
6. Fiscal implications of macroeconomic developments
I. Fiscal Accounts

The Statement of Government Operations – Above the Line

Transactions

**I. Transactions affecting net worth**

1. Revenue (increase net worth)
2. Expense (reduce net worth)

3. Gross Operating Balance = 1 – 2

   2.8 Consumption of fixed capital (depreciation)

4. Net Operating Balance ➔ Change in net worth = 3 – 2.8

**II. Transactions in nonfinancial assets**

5.1 Net acquisition of nonfinancial assets

6. Overall Balance = Net lending/Borrowing = 4 – 5.1
I. Fiscal Accounts

Net operating balance

- Net operating balance represents the change in net worth due to transactions attributable to government policy.
- Focus: the “activity” side of government (by considering transactions in revenue and expense)
- With a positive balance, government can acquire assets or decrease liabilities.
- With a negative balance, the government must incur liabilities or liquidate assets.

A. Net lending and borrowing

- Net Lending/Borrowing (NL/B) is a summary measure indicating the extent to which government is either putting financial resources at the disposal of other sectors in the economy or using the financial resources generated by other sectors.
- Focus: on the “financing” side (by considering transactions in financial assets and liabilities).
- NL/B as an indicator of the financial impact of government activity on the rest of the economy.
## I. Fiscal Accounts

### The Statement of Government Operations – Above and Below the Line Transactions

#### I. Transactions affecting net worth

1. Revenue (increase net worth)
2. Expense (reduce net worth)

<table>
<thead>
<tr>
<th>3. Gross Operating Balance</th>
<th>= 1 – 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.8 Consumption of fixed capital (depreciation)</td>
<td></td>
</tr>
</tbody>
</table>

| 4. Net Operating Balance | Change in net worth = 3 – 2.8 |

#### II. Transactions in nonfinancial assets

| 5.1 Net acquisition of nonfinancial assets |

#### III. Transactions in financial assets and liabilities

| 5.2 Net acquisition of financial assets |
| 5.3 Net incurrence of liabilities |
| 5.3.1 Foreign |
| 5.3.2 Domestic |

| 6. Net lending/Borrowing | = 4 – 5.1 |

### I. Fiscal Accounts

“Above” and “Below” the line

- **the line** refers to whether an item is recorded as part of the overall balance, or instead as financing
- **Revenue** (taxes, profits, and grants) are recorded “above the line”
- **Expense and acquisition of non-financial asset** are “above the line”
- **Repayable resources** (borrowing from markets or from international financial institutions) are treated as financing and are “below the line”.
II. Forecasting interest payments, borrowing and debt

The Simultaneity Problem

Simultaneity problem:
solve with iterative process:
1. Make an initial estimate of interest payments
2. Compute overall balance and new borrowing requirements
3. Recalculate interest payments and start again
II. Forecasting interest payments, borrowing and debt

A simple example

<table>
<thead>
<tr>
<th></th>
<th>2014 Q1</th>
<th>2014 Q2</th>
<th>2014 Q3</th>
<th>2014 Q4</th>
<th>2015 Q1</th>
<th>2015 Q2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disbursement (4% interest, quarterly payments)</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest Payment</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Disbursement (10% interest, semi-annual payments)</td>
<td>140</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest Payment</td>
<td>0</td>
<td>7</td>
<td>0</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Interest payment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Ideally, forecast interest payment on each new issuance separately.

Example 1

- Debt at end 2013 = 1,000; Interest rate on debt = 8% (paid quarterly)
- Primary Balance = -200 (equally divided every quarter)

<table>
<thead>
<tr>
<th></th>
<th>2014Q1</th>
<th>2014Q2</th>
<th>2014Q3</th>
<th>2014Q4</th>
<th>2014</th>
</tr>
</thead>
<tbody>
<tr>
<td>Round 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Primary Balance</td>
<td>-50</td>
<td>-50</td>
<td>-50</td>
<td>-50</td>
<td>-200</td>
</tr>
<tr>
<td>Interest on old debt</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>80</td>
</tr>
<tr>
<td>Overall Balance</td>
<td>-70</td>
<td>-70</td>
<td>-70</td>
<td>-70</td>
<td>-280</td>
</tr>
<tr>
<td>Gross Borrowing</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>280</td>
</tr>
<tr>
<td>Debt stock</td>
<td>1280</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Round 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest on new debt (10%)</td>
<td>0</td>
<td>1.75</td>
<td>3.5</td>
<td>5.25</td>
<td>10.5</td>
</tr>
<tr>
<td>New Overall Balance</td>
<td>-70</td>
<td>-71.75</td>
<td>-73.5</td>
<td>-75.25</td>
<td>-290.5</td>
</tr>
<tr>
<td>Additional Borrowing</td>
<td>0</td>
<td>1.75</td>
<td>3.5</td>
<td>5.25</td>
<td>10.5</td>
</tr>
<tr>
<td>Revised Debt Stock</td>
<td>1290.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Round 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest on new debt (10%)</td>
<td>0</td>
<td>0</td>
<td>0.044</td>
<td>0.131</td>
<td>0.175</td>
</tr>
<tr>
<td>New Overall Balance</td>
<td>-70.00</td>
<td>-71.75</td>
<td>-73.54</td>
<td>-75.38</td>
<td>-290.68</td>
</tr>
<tr>
<td>Additional Borrowing</td>
<td>0</td>
<td>0</td>
<td>0.044</td>
<td>0.131</td>
<td>0.175</td>
</tr>
<tr>
<td>Revised Debt Stock</td>
<td>1290.675</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
II. Forecasting interest payments, borrowing and debt

Example 2: Including amortization

- Now assume, of the old debt stock, that 100 is amortized with the proceeds of a public asset in 2014Q2

<table>
<thead>
<tr>
<th></th>
<th>2014Q1</th>
<th>2014Q2</th>
<th>2014Q3</th>
<th>2014Q4</th>
<th>2014</th>
</tr>
</thead>
<tbody>
<tr>
<td>Round 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Primary Balance</td>
<td>-50</td>
<td>-50</td>
<td>-50</td>
<td>-50</td>
<td>-200</td>
</tr>
<tr>
<td>Interest on old debt</td>
<td>20</td>
<td>20</td>
<td>18</td>
<td>18</td>
<td>76</td>
</tr>
<tr>
<td>Overall Balance</td>
<td>-70</td>
<td>-70</td>
<td>-68</td>
<td>-68</td>
<td>-276</td>
</tr>
<tr>
<td>Gross Borrowing</td>
<td>70</td>
<td>70</td>
<td>68</td>
<td>68</td>
<td>276</td>
</tr>
<tr>
<td><strong>Debt stock</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>1176</strong></td>
</tr>
</tbody>
</table>

Round 2

- Interest on new debt (10%) | 0 | 1.75 | 3.5 | 5.2 | 10.45 |
- New Overall Balance | -70 | -71.75 | -71.5 | -73.2 | -286.45 |
- Additional Borrowing | 0 | 1.75 | 3.5 | 5.2 | 10.45 |
- **Revised Debt Stock** |        |        |        |        | **1186.45** |

Round 3

- Interest on new debt (10%) | 0 | 0 | 0.044 | 0.131 | 0.175 |
- New Overall Balance | -70.00 | -71.75 | -71.54 | -73.33 | -286.63 |
- Additional Borrowing | 0 | 0 | 0.044 | 0.131 | 0.175 |
- **Revised Debt Stock** |        |        |        |        | **1186.625** |

II. Forecasting interest payments, borrowing and debt

Example 3: Including amortization with new issuance

- Now assume, of the old debt stock, that 100 is amortized in 2014Q2 through the issuance of new debt with interest rate of 10%

<table>
<thead>
<tr>
<th></th>
<th>2014Q1</th>
<th>2014Q2</th>
<th>2014Q3</th>
<th>2014Q4</th>
<th>2014</th>
</tr>
</thead>
<tbody>
<tr>
<td>Round 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Primary Balance</td>
<td>-50</td>
<td>-50</td>
<td>-50</td>
<td>-50</td>
<td>-200</td>
</tr>
<tr>
<td>Interest on old debt</td>
<td>20</td>
<td>20</td>
<td>20.5</td>
<td>20.5</td>
<td>81</td>
</tr>
<tr>
<td>Overall Balance</td>
<td>-70</td>
<td>-70</td>
<td>-70.5</td>
<td>-70.5</td>
<td>-281</td>
</tr>
<tr>
<td>Gross Borrowing</td>
<td>70</td>
<td>70</td>
<td>70.5</td>
<td>70.5</td>
<td>281</td>
</tr>
<tr>
<td><strong>Debt stock</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td><strong>1281</strong></td>
</tr>
</tbody>
</table>

Round 2

- Interest on new debt (10%) | 0 | 1.75 | 3.5 | 5.26 | 10.51 |
- New Overall Balance | -70.00 | -71.75 | -74   | -75.76 | -291.51 |
- Additional Borrowing | 0 | 1.75 | 3.5 | 5.26 | 10.51 |
- **Revised Debt Stock** |        |        |        |        | **1291.51** |

Round 3

- Interest on new debt (10%) | 0 | 0 | 0.044 | 0.131 | 0.18 |
- New Overall Balance | -70.00 | -71.75 | -74.04 | -75.89 | -291.69 |
- Additional Borrowing | 0 | 0 | 0.044 | 0.131 | 0.18 |
- **Revised Debt Stock** |        |        |        |        | **1291.69** |
II. Forecasting interest payments, borrowing and debt

A simpler method

A valid shortcut consists of assuming that new borrowing is incurred at mid-year (Denote: \( I = \text{Total Interest Payment} \); \( i \) is interest rate)

\[
I_t = i_t^{old} D_{t-1} + i_t^{new} \frac{\text{new borrowing}}{2}
\]

- If \( i_t^{old} \neq i_t^{new} \) compute the “implicit” interest rate so that:

\[
I_t = i_t^* \left[ D_{t-1} + \frac{D_t - D_{t-1}}{2} \right] = i_t^* \left[ \frac{D_t + D_{t-1}}{2} \right]
\]

- For \( I_{t+1} \), we need to make projections of the future implicit interest rate.

II. Forecasting interest payments, borrowing and debt

Forecasting interest rate on future borrowing

Steps:

1. Check historical trend of past interest rate on government debt relative to some benchmark (central bank policy rate, international rate—LIBOR, US T-bill rate...)

2. Consider if there is likely to be a change in the risk premium (spread over the benchmark) on future debt issuances

3. Make projections of the benchmark rate and the risk premium, and hence the future rate on government debt

The inertia of future implicit interest rate depends on the amount of debt maturing next year
### II. Forecasting interest payments, borrowing and debt

#### Forecasting implicit interest rate – example 1

<table>
<thead>
<tr>
<th></th>
<th>2014</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock of debt outstanding at year end</td>
<td>1000</td>
<td>1200</td>
</tr>
<tr>
<td>Amount maturing in mid-2016</td>
<td>...</td>
<td>200</td>
</tr>
<tr>
<td>New borrowing</td>
<td>...</td>
<td>400</td>
</tr>
<tr>
<td>Implicit interest rate on old debt stock</td>
<td>5%</td>
<td>...</td>
</tr>
<tr>
<td>Interest payment on old debt stock</td>
<td>...</td>
<td>$45 = 5% \times 800 + 2.5% \times 200$</td>
</tr>
<tr>
<td>Interest rate on new borrowing</td>
<td>...</td>
<td>20%</td>
</tr>
<tr>
<td>Interest payment on new borrowing</td>
<td>...</td>
<td>$40 = 10% \times 400$</td>
</tr>
<tr>
<td>Total interest payments</td>
<td>...</td>
<td>95</td>
</tr>
<tr>
<td>Average stock of debt</td>
<td>...</td>
<td>1100</td>
</tr>
<tr>
<td>Implicit interest rate on new debt stock</td>
<td>...</td>
<td>$8.6%$</td>
</tr>
</tbody>
</table>

### II. Forecasting interest payments, borrowing and debt

#### Forecasting implicit interest rate – example 2

<table>
<thead>
<tr>
<th></th>
<th>2014</th>
<th>2015</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock of debt outstanding at year end</td>
<td>1000</td>
<td>1200</td>
</tr>
<tr>
<td>Amount maturing in mid-2015</td>
<td>...</td>
<td>800</td>
</tr>
<tr>
<td>New borrowing</td>
<td>...</td>
<td>1400</td>
</tr>
<tr>
<td>Interest rate on old debt stock</td>
<td>5%</td>
<td>...</td>
</tr>
<tr>
<td>Interest payment on old debt stock</td>
<td>...</td>
<td>$20 = 5% \times 200 + 2.5% \times 800$</td>
</tr>
<tr>
<td>Interest rate on new borrowing</td>
<td>...</td>
<td>20%</td>
</tr>
<tr>
<td>Interest payment on new borrowing</td>
<td>...</td>
<td>$140 = 10% \times 1400$</td>
</tr>
<tr>
<td>Total interest payments</td>
<td>...</td>
<td>160</td>
</tr>
<tr>
<td>Average stock of debt</td>
<td>...</td>
<td>1100</td>
</tr>
<tr>
<td>Implicit interest rate on new debt stock</td>
<td>...</td>
<td>$14.5%$</td>
</tr>
</tbody>
</table>
### III. Debt Dynamics

#### Accumulation of New Debt

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public debt ( t ) - Public debt ( t-1 )</td>
<td>( = ) Interest rate ( \times ) ( \frac{\text{Public Debt}_t}{\text{Public Debt}_t} ) - Primary balance (PB) + Other Flows</td>
</tr>
<tr>
<td>Public debt ( t ) - Public debt ( t-1 )</td>
<td>( = ) - Net lending/borrowing + Other flows</td>
</tr>
<tr>
<td>Overall balance</td>
<td>E.g. Privatization receipts</td>
</tr>
</tbody>
</table>

\( t \) denotes the current period, and \( t-1 \) denotes the previous period.
II. Public Debt Dynamics

What are the “other flows”?

Flows reducing debt stock

- Exceptional Financing: Debt reduction (MDRI, bilateral debt relief)
- Privatization receipts
- Asset valuation (exchange rate effects)

Flows increasing debt stock

- Contingent liabilities
- Asset valuation (exchange rate effects)

III. Public Debt Dynamics

Notation (ignoring external debt)

\[ D_t \] stock of debt at time \( t \)
\[ PB_t \] primary or non-interest surplus at time \( t \)
\[ O_t \] Other Flows at time \( t \)
\[ r_t \] real interest rate at time \( t \)
\[ \pi_t \] inflation rate at time \( t \)
\[ i_t \] nominal interest rate at time \( t \)
\[ g_t \] real GDP growth rate at time \( t \)
\[ P_t Y_t \] nominal GDP at time \( t \)

\[ \Delta D_t = iD_{t-1} - PB_t + O_t \]
\[ 1 + i_t = (1 + \pi_t)(1 + \pi_t) \]
\[ P_t Y_t = (1 + \pi_t)(1 + g_t)P_{t-1}Y_{t-1} \]
III. Public Debt Dynamics
Law of motion of debt and ignoring other flows

- Government Debt at time $t$
  \[ D_t = (1+i_t)D_{t-1} - PB_t \]

- Divide equation above by $P_t Y_t$ (nominal GDP):
  \[ \frac{D_t}{P_t Y_t} = \frac{(1+i_t)}{1+\pi_t}(1+g_t)\frac{D_{t-1}}{P_{t-1} Y_{t-1}} - \frac{P Y_t}{P_{t-1} Y_{t-1}} \]
  where $\pi$ is inflation; $g$ is real GDP growth

- Note that:
  \[ \phi_t = \frac{1+i_t}{(1+\pi_t)(1+g_t)} = \frac{1+r_t}{1+g_t} \]
  where $r$ is the real interest rate

- Key equation
  \[ d_t = \phi_t d_{t-1} - pb_t \]

  If $\phi_t < 1$ (i.e. $r_t < g_t$) then $d_t$ converges, and debt is sustainable
  If $\phi_t > 1$ (i.e. $r_t > g_t$), then $d_t$ explodes

IV. Ways of financing the budget
Financing the budget: forms

The way in which the deficit is financed has macroeconomic consequences.

There are four forms of financing:
• Borrowing from the central bank
• Borrowing from the domestic commercial banks
• Borrowing from the domestic nonbank sector
• Borrowing abroad

Financing the budget

<table>
<thead>
<tr>
<th>Total financing = - overall balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreign (net)</td>
</tr>
<tr>
<td>New borrowing</td>
</tr>
<tr>
<td>Repayments</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Domestic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank lending (net) – including central bank</td>
</tr>
<tr>
<td>New borrowing</td>
</tr>
<tr>
<td>Repayments of debt</td>
</tr>
<tr>
<td>Deposits</td>
</tr>
<tr>
<td>Non-Bank (net)</td>
</tr>
<tr>
<td>Privatization receipts</td>
</tr>
</tbody>
</table>
Financing the budget

- Deposits at the central bank are an asset: accumulating deposits is an increase in assets (= decrease in net debt), drawing-down on deposits is a decrease in assets (= increase in net debt).
- Borrowing abroad (or domestically) to increase deposits is neutral on net debt (why?)

V. Macro implications of budget financing
Financing the budget

- Central bank borrowing: by expanding money supply, can add to inflation. But monetary expansion can also increase the impact of fiscal expansion on demand.
- Commercial bank borrowing: crowds out private borrowing, raising interest rates, unless central bank accommodates by supplying reserves. Then like above.
- Domestic non-bank borrowing: crowds out private borrowing that could be used to finance investment, less funding available for loans leads to higher interest rates
- Foreign borrowing: raises foreign debt and may lead to Balance of Payment problems; exchange rate risk; debt service needs may exert downward pressure on the exchange rate

The type and amount of public debt has also consequences on:
- Macroeconomic environment
- Policy space
- Financial sector stability
Financing the Budget: Balance sheet connections across sectors (I)

- Consider the stock of assets and liabilities at a certain point in time for the main sectors of the economy:
  - the government (including the central bank)
  - the private financial sector (mainly banks)
  - the non-financial sector (corporations and households)
  - external (non-resident) entities

---

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
</table>

---

<table>
<thead>
<tr>
<th>Government sector</th>
<th>Financial sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets</td>
<td>Liabilities</td>
</tr>
<tr>
<td>Claims on:</td>
<td>Obligations to:</td>
</tr>
<tr>
<td>Financial sector</td>
<td>Government sector</td>
</tr>
<tr>
<td>Non-fin private sector</td>
<td>Financial sector</td>
</tr>
<tr>
<td>External</td>
<td>Non-fin private sector</td>
</tr>
<tr>
<td></td>
<td>External</td>
</tr>
<tr>
<td></td>
<td>Net worth</td>
</tr>
</tbody>
</table>

---

<table>
<thead>
<tr>
<th>Non-financial private sector</th>
<th>External sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assets</td>
<td>Liabilities</td>
</tr>
<tr>
<td>Claims on:</td>
<td>Obligations to:</td>
</tr>
<tr>
<td>Government</td>
<td>Government</td>
</tr>
<tr>
<td>Financial sector</td>
<td>Financial sector</td>
</tr>
<tr>
<td>External</td>
<td>Non-fin private sector</td>
</tr>
<tr>
<td></td>
<td>External</td>
</tr>
<tr>
<td></td>
<td>Net worth</td>
</tr>
</tbody>
</table>
VI. Fiscal effects of macroeconomic developments

• During an economic contraction revenues-to-GDP fall and expenditure-to-GDP increases
• Despite high growth, fall in commodity prices can cause revenues-to-GDP to fall
• In good times higher revenues may trigger higher discretionary expenditures i.e. induce pro-cyclicality and allocation of expenditures may not be optimal
Inflation and the budget

- Increase in commodity prices and cost of subsidies
- Higher inflation rate could lead to higher nominal interest rates and higher interest spending in government budget
- Higher inflation rate would lead to appreciation of currency in real terms and loss of competitiveness
- Very high inflation could lead to uncertainty, decline in investments, and thus GDP and revenues

Real interest rates and the budget

- Lower interest rates mean lower interest payments
- Lower interest rates could spur investment and higher GDP growth; also corporate costs decline leading to higher corporate profits and tax collections
- Low foreign interest rates – search for yield could lead to capital inflows
- High capital inflows could lead to more liquidity supporting higher growth as well as lower interest rates (maybe higher inflation) and higher revenues
Real exchange rate and the budget

- If the Real Effective Exchange Rate (REER) appreciates, lose competitiveness lowering GDP:
  - Exports decline
  - Cheaper imports hurt domestic industry
- High external debt – if exchange rate depreciates or global interest rates rise, liability increases as do interest payments; if not rolled over could cause capital flight and lower growth and revenues
- Real appreciation may lead to capital flows and higher revenues on capital gains

Development, demographics and the budget

- As an economy develops the government can rely on a more diverse tax base (adoption of technology makes it easier to comply, to better target beneficiaries of exemption, to improve tax administration...)
- Richer economies can usually support more extensive social security and safety nets and have a larger government
- Age structure affects mix of expenditure (if young population, need to spend more on their education; an aging population requires more spending on health care and public pension; also, revenue growth may also be declining with large old population)
Thank you
Outline of the lecture

1. Fiscal Aggregates
2. Adjusted Balances
3. Fiscal Position

I. Fiscal Aggregates
Introduction

• Activities of the government sector
• Collect revenue
• Produce goods and services
• Redistribute income via transfers

• Activities lead to financing to and from the rest of the economy.

Questions:
What is the government’s macroeconomic impact?
Is government activity sustainable over time?

Introduction

Why use fiscal indicators

• Fiscal indicators shed light on:
  • How does the fiscal sector impact domestic demand?
    • Expenditure: changes in expenditure affect aggregate demand
    • Revenue: changes in revenue affect aggregate demand
  • What is the stance of fiscal policy?
    • What is the underlying fiscal position?
    • Is discretionary fiscal policy expansionary or contractionary?
  • Is public debt sustainable?
## I. Fiscal Aggregates

### Different uses of fiscal indicators

<table>
<thead>
<tr>
<th>I. Transactions affecting net worth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Revenue (increase net worth)</td>
</tr>
<tr>
<td>2. Expense (reduce net worth)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>III. Transactions in financial assets and liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.2 Net acquisition of financial assets</td>
</tr>
<tr>
<td>5.3 Net incurrence of liabilities</td>
</tr>
</tbody>
</table>

### II. Transactions in nonfinancial assets

| 5.1 Net acquisition of nonfinancial assets           |
| 5.1.1 Fixed assets                                  |
| 5.1.2 Change in inventories                         |
| 5.1.3 Other                                        |

<table>
<thead>
<tr>
<th>6. Net lending/Borrowing</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 – 5.1</td>
</tr>
</tbody>
</table>

### Net Operating Balance

\[ \text{Net Operating Balance} = \text{Revenue} - \text{Expense} - \text{Consumption of fixed capital} \]

\[ = 1 - 2 - 0.8 \]

\[ = 3 - 2.8 \]

<table>
<thead>
<tr>
<th>2.8 Consumption of fixed capital (depreciation)</th>
</tr>
</thead>
</table>
I. Fiscal Aggregates

Net operating balance

- Net operating balance represents the change in net worth due to transactions attributable to government policy.
- **Focus**: the “activity” side of government (by considering transactions in revenue and expense)
- With a positive balance, government can acquire assets (financial or non-financial) or decrease liabilities.
- With a negative balance, the government must incur liabilities or liquidate assets (financial or non-financial)

I. Fiscal Aggregates

Net lending and borrowing

- Net Lending/Borrowing (NL/B) is a summary measure indicating the extent to which government is either putting financial resources at the disposal of other sectors in the economy or using the financial resources generated by other sectors.
- **Focus**: on the “financing” side (by considering transactions in financial assets and liabilities).
- NL/B as an indicator of the financial impact of government activity on the rest of the economy.
I. Fiscal Aggregates

Primary balance

Definition: Primary balance = overall balance + interest payments

Rationale:
• An indicator of current fiscal effort, since interest payments are predetermined by the size of previous deficits
• The primary balance is a key variable used in debt-sustainability analysis.

I. Fiscal Aggregates

Non-resource balances

• Resource revenue: Revenue receivable that is related to natural resources. These receivables may be related to various types of taxes, subsidies, dividends, contracts, leases, and licenses, rent, or other transfers.
• Resource expense: Expense payable that is related to natural resources. These payables may be related to various types of expense such as subsidies, property expense, and transfers.

• Non-resource balance: overall balance excluding net resource revenue. A measure of fiscal stance
• Non-resource primary balance: primary balance excluding net resource revenue. A measure of fiscal sustainability

Rationale: resource revenue are volatile and exhaustible
II. Adjusted Fiscal Balances

Cyclically-adjusted balance

Definition: The cyclically-adjusted balance (CAB) “strips” the cyclical effects from the overall balance. Both revenues and expenditures (e.g. income and sales tax revenues, unemployment insurance outlays) are adjusted.

Cyclical adjustment corrects for variations in fiscal revenue and expenditure due to the business cycle.

Rationale: We should adjust the overall fiscal balance to account for factors that cannot be imputed to government action, but are attributable to business cycle
II. Adjusted Fiscal Balances

Why cyclical adjustment?

• Revenue, expense and overall balance could be misleading measures of the fiscal policy stance.

• Some fiscal variables respond automatically to temporary changes in output.

• For example, when the output gap is negative:

  • Unemployment benefits are higher than usual.
  • Household and corporate incomes are lower and, therefore, most tax revenues are lower. Same goes for HH consumption and VAT.
  • Revenues from taxes on capital gains and property are also likely to be lower.
  • If the tax system is progressive, tax revenues will be lower still.

  automatic stabilizers

II. Adjusted Fiscal Balances

Cyclically-adjusted balance: the Importance of Automatic Stabilizers

• The most important factor determining the cyclical sensitivity of the fiscal position is the size of the general government sector.

• Tax structure is also important: The greater the taxation of cyclically-sensitive tax bases, the more cyclical revenues will be.

• Other factors: the generosity of transfer incomes and the progressivity of the tax system.
II. Adjusted Fiscal Balances

Procedure for cyclical adjustment

\[ \text{CAB} = \text{Overall balance} - \text{cyclical component of the balance} \]

Obtaining cyclical component requires:
- An estimate of the **output gap**
- Elasticities of revenue/expenditure to the output gap
- Difference between actual and potential output (a measure of business cycles)
  \[ \frac{Y - Y^*}{Y^*} \]

Often expressed as percentage of potential output.
II. Adjusted Fiscal Balances  
Estimating Potential Output

- Time series techniques (IMF offers MF course):
  - Linear regression
  - Hodrick-Prescott (HP) filter
- Production function approach
- There is no formal test that separates good estimates from bad ones


---

II. Adjusted Fiscal Balances  
Revenue and the Cycle

\[ R_i = \hat{R}_i \left( \frac{Y}{Y^*} \right)^{\varepsilon_i} \]

Nominal revenues (total or itemized)
Actual GDP
Elasticity
Adjusted revenues: nominal revenues (total or itemized) that would prevail with no output gap
Potential GDP

Is \( \varepsilon_i \) smaller or greater than zero?
\( \varepsilon_i > 0 \): revenues are procyclical
Note: Here, \( (Y/Y^*) \) represents the output gap.
II. Adjusted Fiscal Balances

Expenditure and the Cycle

\[ G_i = G_i^\ast \left( \frac{Y}{Y^\ast} \right)^\eta \]

- **Adjusted expenditures**: Nominal expenditures (total or itemized) that would prevail with no output gap
- Actual GDP
- Elasticity
- Potential GDP
- Note: Here, \( \frac{Y}{Y^\ast} \) represents the output gap.

Step 1: Compute the revenues and expenditures that would have prevailed if output had been at potential

\[ R_i^\ast = R_i \left( \frac{Y^\ast}{Y} \right)^\varepsilon \]
\[ G_i^\ast = G_i \left( \frac{Y^\ast}{Y} \right)^\eta \]

Step 1.b: If itemized, sum to obtain total adjusted revenues and expenditures

Step 2: Compute the CAB in percent of potential output

\[ cab^\ast = \frac{R^\ast - G^\ast}{Y^\ast} \]

Revenue and expenditure items excluded from the adjustment (for example, interest payments)
II. Adjusted Fiscal Balances
CAB and Actual Overall Balance

Recall that:

\[ R^* = R \left( \frac{Y^*}{Y} \right)^{\varepsilon_i} \quad \varepsilon_i > 0 \]
\[ G_i^* = G_i \left( \frac{Y^*}{Y} \right)^{\eta_i} \quad \eta_i \leq 0 \]

**Boom:**
\[ Y > Y^* \implies R > R^*, G \leq G^* \]
CAB < Actual Balance
Public finances (actual b) are in better shape than they would be (CAB) because economy is in a boom.

**Recession:**
\[ Y < Y^* \implies R^*, R, G^* \leq G \]
CAB > Actual Balance
Public finances would have been in better shape, if the economy were not in a recession.

II. Adjusted Fiscal Balances
CAB: Aggregated vs. Disaggregated Approach

- **Aggregate approach:**
  - Same elasticity for all revenues and same elasticities for all expenditures elasticities can be
  - common values are 1 for revenue and 0 for expenditure
  - Source: from the literature or estimated

- **Disaggregated approach (OECD methodology):**
  - Consider different elasticities for **disaggregated** categories of revenue and expenditure
    - Corporate income tax
    - Personal income tax
    - Indirect taxes (VAT, import duties, excises)
    - Social security contributions
II. Adjusted Fiscal Balances
CAB: Aggregated vs. Disaggregated Approach

**Aggregated approach**

- Business cycle
- Cyclic revenue (elasticity > 1)
- Cyclic expenditure (elasticity = 0)

**Disaggregated approach**

- Business cycle
- Cyclic income tax receipts
- Cyclic VAT receipts
- Cyclic unemployment expenditure

- Cyclic revenue (sum)

Reference: IMF Technical Note and Manuals 11/02

---

II. Adjusted Fiscal Balances
Disaggregated Elasticities – some country examples

<table>
<thead>
<tr>
<th>Country</th>
<th>Corporate Tax</th>
<th>Personal Tax</th>
<th>Indirect Taxes</th>
<th>Social Security Contributions</th>
<th>Primary Current Expenditure</th>
<th>Overall Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Japan</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
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<tr>
<td>Italy</td>
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- EC uses these estimates in their budgetary surveillance ([EC Paper (2005)](EC Paper (2005)))
II. Adjusted Fiscal Balances
Disaggregated Elasticities – how to calculate your own

• In general, 

\[ E_i = E_{i, \text{tax base}} \frac{Y}{Y_i} \]

Elasticity of tax revenues to its tax base

Elasticity of tax base to output gap

How to calculate the elasticities:
1. Tax revenue to its tax base: regress the log of tax base on log of tax revenue
2. Tax base to output: regress the log of output gap on log of tax base

<table>
<thead>
<tr>
<th>TABLE 1. COMMON TAX ELASTICITIES</th>
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<tbody>
<tr>
<td>Tax category</td>
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<tr>
<td>Personal income taxes</td>
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<td>Corporate income taxes</td>
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<tr>
<td>Social security contributions</td>
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<tr>
<td>Indirect taxes</td>
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</tbody>
</table>

Source: Girouard and André (2005).

II. Adjusted Fiscal Balances
CAB: Finding elasticities - evidence from sub-Saharan Africa

• Elasticity of revenue to GDP
  • Revenue = Taxes + Non-tax revenue + Grants
    • Most taxes are in the form of indirect taxes. Indirect taxes are pro-cyclical, with elasticity of revenue to potential GDP is close to 1
    • Non-tax revenue are small relative to total revenue
    • Foreign grants are important in many countries and they tend to be acyclical, i.e. elasticity close to 0.
  • Elasticity of expenditure to GDP
    • Expenditure = current + capital + transfer payment
      • Most of expenditures are not related to transfer payments
      • Current and capital expenditure are acyclical, i.e. elasticity close to 0.
II. Adjusted Fiscal Balances

**Structural balance**

- Structural adjustment also corrects for the impact of one-off operations, and other transient influences (asset and commodity prices, output composition effects, etc.) beyond the output cycle.
- The structural balance facilitates the separation of fiscal balances into discretionary and non-discretionary parts

**Rationale:** We need to adjust the fiscal balance to account for factors that cannot be imputed to government action, but can be attributable to business cycle, commodity price shocks and other one-off events.

**II. Adjusted Fiscal Balances**

**Structural Balance - Steps**

Structural balance is the overall balance adjusted for a broader range of exogenous factors. It is broader than the CAB as it includes one-off factors and other asset price changes (e.g. commodity prices).

**Procedure (3 steps)**

**Step 1:** Identify and remove one-off fiscal operations

**Step 2:** Adjust for effects of business cycles

**Step 3:** Adjust for effects of other factors (asset prices, etc.)

- Can be viewed as an *augmentation* of CAB
- Can be a better measure of *underlying fiscal balance* if effects of other cycles and factors are significant
### III. Fiscal Position

#### Fiscal Stance

Quantifies how much fiscal policy add to or subtract from **domestic demand**

- Expansionary (or loose)
- Contractionary (or tight)

Definition, using CAB:

\[ FS_t = -CAB_t \]

- \( CAB_t < 0 \) \( \Rightarrow \) Expansionary \( (FS_t > 0) \)
- \( CAB_t = 0 \) \( \Rightarrow \) Neutral \( (FS_t > 0) \)
- \( CAB_t > 0 \) \( \Rightarrow \) Contractionary \( (FS_t < 0) \)

- Note: We can use other fiscal indicators to measure fiscal stance. E.g. Non-resource primary balance or structural balance
Fiscal impulse measures the change in fiscal stance over time:

\[ FI = FS_t - FS_{t-1} \]

### III. Fiscal Position

Fiscal Impulse

The fiscal impulse – an example

Question: how has the fiscal impulse changed from 2011 to 2010 in Asia?

**Figure 1.9. Asia: Fiscal Impulse**

*(In percent of GDP)*

Source: IMF staff estimates.

1 Based on annual change (calendar year) in general government cyclically adjusted fiscal balance to GDP ratios. A negative number implies withdrawal of fiscal stimulus.

Source: REO APD Apr 2011
Does an expansionary fiscal impulse actually lead to an economic expansion?

It depends on:

- **Expectations** by the private sector
- How the deficit is **financed**
  - e.g. temporary stimulus followed by an increase in tax
- **Composition** of the change in fiscal policy
  - Note: Fiscal stance and impulse measures implicitly assume that the impact of different instruments are the same.