East AFRITAC
INTERNATIONAL MONETARY FUND
REGIONAL TECHNICAL ASSISTANCE CENTER

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REGIONAL TRAINING ON MACRO-FISCAL FORECASTING

ARUSHA, AUGUST 8 – 18, 2016

Day 1: Monday August 8

Time	Subject	Resource Person
08:30 – 09:00	Registration and Introduction	
09:00 – 09:30	Quiz	
09:30 – 10:15	L1. Overview of Macroeconomic Forecasting	Fazeer Rahim
10:15 – 10:45	Coffee break and Group Photo	
10:45 – 12:30	W1. Introduction to Forecasting Using EViews	Fazeer Rahim
12:30 – 13:30	Lunch Break	
13:30 – 14:30	L2. Properties of Time Series Data I: Stationarity, Box Jenkins ARIMA Models	Heloisa Marone
15:00 – 15:30	Coffee break	
14:30 – 17:00	W2. Estimating and Forecasting ARIMA Models using EViews	Heloisa Marone

Day 2: Tuesday August 9

Time	Subject	Resource Person
08:30 – 09:30	L3. Properties of Time Series Data II: Nonstationarity and Unit Roots	Heloisa Marone
09:30 – 10:00	Coffee break	
10:00 – 12:30	W3. Detecting Nonstationary Time Series in Practice	Heloisa Marone
12:30 – 13:30	Lunch Break	
13:30 – 14:30	L4. Cointegration I—Single Equation Estimation, Error Correction Models and Forecasting	Fazeer Rahim
14:30 – 15:00	Coffee break	
15:30 – 16:30	W4. Forecasting using Single Equation Estimation Methods and ECM Models	Fazeer Rahim

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Day 3: Wednesday August 10

Time	Subject	Resource Person
08:30 – 10:15	L5. Vector Autoregression (VAR), Structural VAR Models, Impulse Response Functions (IRFs)	Fazeer Rahim
10:15 – 10:45	Coffee break	
10:45 – 12:30	W5. VAR and IRF Application: UK Money Demand—Part I	Fazeer Rahim
13:00 – 14:00	Lunch Break	
14:00 – 14:45	L6. Cointegration II: Johansen Methodology	Heloisa Marone
16:00 – 16:30	Coffee break	
16:30 – 17:30	W6. UK Money Demand—Part II	Heloisa Marone

Day 4: Thursday August 11

Time	Subject	Resource Person
08:30 – 09:30	L7. Vector Error Correction Models: Formulation, Hypothesis Testing, and Forecasting	Heloisa Marone
09:30 – 10:00	Coffee break	
10:00 – 12:30	W7. Forecasting using a VECM	Heloisa Marone
13:00 – 14:00	Lunch Break	
14:00 – 17:00	Preparation for participants' presentation	

Day 5: Friday August 12

Time	Subject	Resource Person
09:00– 10:00	L8. Evaluating regression models	Phyllis Resnick
10:30 – 11:00	Coffee break	
11:00 – 12:30	W8. Evaluating regression models	Phyllis Resnick
12:30 – 13:30	Lunch Break	
13:30 – 16:30	Participants' presentations	

Day 6: Monday August 15

Time	Subject	Resource Person
09:00 – 10:15	L9. Elements of Revenue Forecasting I: Macroeconomic Assumptions and the Effective Tax Rate Approach	Fazeer Rahim
10:30 – 11:00	Coffee break	
10:45 – 12:30	W9. Revenue Forecast	Fazeer Rahim
12:30 – 13:30	Lunch Break	
13:30 – 14:30	L10. Elements of Revenue Forecasting II: the Elasticity Approach and Projections of Revenue Components	Phyllis Resnick
14:30 – 17:00	W10. Revenue Forecast II	Phyllis Resnick

Day 7: Tuesday August 16

Time	Subject	Resource Person
09:00 – 12:30	L11. Expenditure Forecasting	Phyllis Resnick
12:30 – 13:30	Lunch Break	
13:30 – 16:30	W11. Expenditure Forecasting	Phyllis Resnick

Day 8: Wednesday August 17

Time	Subject	Resource Person
09:00 – 10:30	L12. Overall balance, net lending/borrowing and debt, financing the budget and monetary policy	Phyllis Resnick
10:30 – 11:00	Coffee break	
11:00 – 12:30	W13. Completing the budget forecast and projecting debt	Phyllis Resnick
12:30 – 13:30	Lunch Break	
13:30 – 14:30	L14. Analysis of Fiscal Aggregates, Adjusted Balances, and Fiscal Position	Fazeer Rahim
14:30 – 15:00	Coffee break	
15:00 – 17:30	W14. Analysis of Fiscal Stance	Fazeer Rahim

Day 9: Thursday August 18

Time	Subject	Resource Person
09:00 – 11:00	Group presentations and wrap up	
11:00 – 11:30	Coffee break	
11:30 – 13:00	Concluding session	
13:00 – 14:00	Lunch Break	

International Monetary Fund - IMF
Regional Technical Assistance Center
Building Macroeconomic Capacity in East Africa

IMF East AFRITAC

Workshop on Macro-Fiscal Forecasting
Arusha, August 8 – 18, 2016

L1. Overview of Macroeconomic Forecasting

Fazeer Rahim (IMF East AFRITAC)

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Outline

1. Preliminaries
2. Quick overview of forecasting methods
3. Evaluation of forecasts
4. Week 1 – Format of training

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Preliminaries
Choices for Forecasting

- What is the variable of interest? For example, GDP growth or inflation.
- What is the forecast horizon? For example, one quarter or one year.
- Would you like point or interval forecasts? For example, GDP growth next year will be 2% or GDP growth will be between 1% and 3% with a probability of 80%? Latter probably more useful for policy makers, since point forecasts are by definition random variables subject to error.

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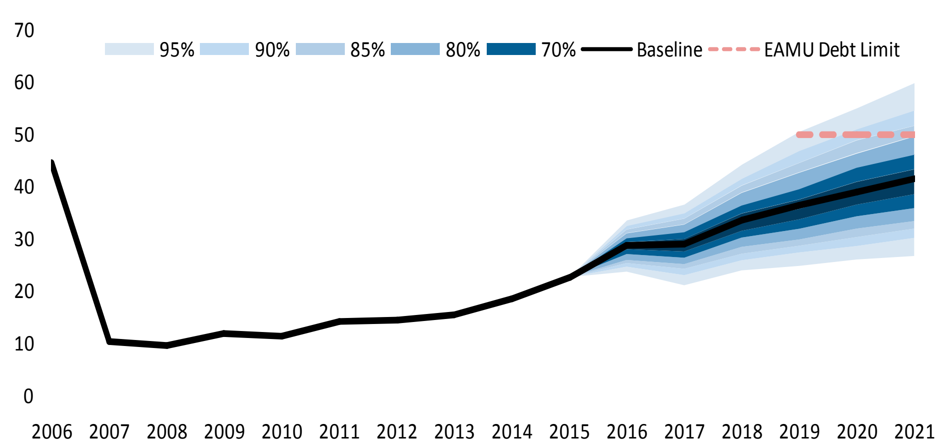
Preliminaries
Example: point vs interval forecast

Uganda: 5-year projection of Uganda's debt to GDP ratio



Preliminaries
Forecast sources

- Informal methods: Surveys, polls, personal judgment
  - DGP (data generation process) is not modeled explicitly
- Econometric models: non-structural and structural
  - Attempt to approximate the true DGP using an econometric model.
- Combination forecasts: combination of informal and econometric approaches

It is advisable to employ several forecasting methods and pool the obtained forecasts

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Preliminaries
Why are forecasts often "wrong"?

- Classic sampling error (the probability that the statistical technique detects the true DGP is zero). And even if it does detect the true DGP, its error term is unpredictable
- The forecasting model is misspecified
- Lucas critique: economic policy and private agents react to existing forecasts (good or bad). In other words, the outcome may depend on the forecast itself ("rational expectations")

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Preliminaries
Forecasting approaches

A. Judgmental approach

B. Time Series Methods

- Naïve (Random walk) model
- Deterministic and stochastic trend models
- ARMA, ARIMA (Box-Jenkins) models
- VAR/VECM models
- Structural econometric models

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Overview of forecasting
Judgmental approach

- Short-run intelligence on recent past, current period and near future
- Can use more data than will work in an econometric model
- Frequently used to forecast exogenous variables
- But hard to explain and justify
- Hard to determine why errors occurred and avoid next time

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Overview
Time series models

1. Naïve (random walk)
2. Deterministic trend
3. Box-Jenkins
4. Vector Autoregression
5. Structural econometric models

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Overview
A. Naïve (Random walk) model

- Forecast is equal to current value

$$y_t = y_{t-1} + u_t \quad u_t \sim NI(0, \sigma^2)$$

$$E_t[y_{t+1}] = y_t + E_t[u_{t+1}]$$
$$= y_t$$

- Often used as a benchmark
- But it is a "Black box" (no economic theory)

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Overview

B. Deterministic trend models

- Fitting linear, quadratic or higher trends
- Example (linear):

$$y_t = \alpha_1 + \alpha_2 t + u_t \quad u_t \sim NI(0, \sigma^2)$$

$$E_t[y_{t+1}] = \alpha_1 + \alpha_2 (t+1) + E_t[u_{t+1}]$$

$$= \alpha_1 + \alpha_2 (t+1)$$

- Another possible benchmark
- It is easy and fast and there is no exogenous variable to forecast
- But it is a "Black box" (no economic theory)

Overview

C. Box-Jenkins (ARIMA) models

- Univariate without exogenous variables: forecast only one variable
- Idea: capture the dynamics of a time series, that is how
  - Past values determine current value (autoregressive component)
  - Past shocks to the variable determine its current value (moving average component)
- Example: ARMA(2,1)

Model

$$y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \beta_1 u_{t-1} + u_t \quad u_t \sim NI(0, \sigma^2)$$

$$E_t[y_{t+1}] = \alpha_1 y_t + \alpha_2 y_{t-1} + \beta_1 u_t + E_t[u_{t+1}]$$

$$= \alpha_1 y_t + \alpha_2 y_{t-1} + \beta_1 u_t$$

Forecast

Overview
C. Box-Jenkins models – pros and cons

- Pros:
  - Easy and fast to implement and estimate
  - No exogenous variable to forecast
- But remains a "Black box" (no economic theory)

Overview
D. Vector Auto-Regression (VAR) models

- Multiple equations (i.e., multiple variables to forecast) without (typically) exogenous variables
- Depends on
  - the past values of all the variables included in the VAR model
- Example: VAR(2,2)

$$y_t = \alpha_{11} y_{t-1} + \alpha_{12} y_{t-2} + \beta_{11} x_{t-1} + \beta_{12} x_{t-2} + u_{1t} \quad u_{1t} \sim NI(0, \sigma_1^2)$$
$$x_t = \alpha_{21} y_{t-1} + \alpha_{22} y_{t-2} + \beta_{21} x_{t-1} + \beta_{22} x_{t-2} + u_{2t} \quad u_{2t} \sim NI(0, \sigma_2^2)$$

Overview
D. Vector Auto-Regression (VAR) models

- Forecast multiple variables simultaneously

$$
\begin{aligned}
E_t[y_{t+1}] &= \alpha_{11}y_t + \alpha_{12}y_{t-1} + \beta_{11}x_t + \beta_{12}x_{t-1} + E_t[u_{1,t+1}] \\
&= \alpha_{11}y_t + \alpha_{12}y_{t-1} + \beta_{11}x_t + \beta_{12}x_{t-1} \\
E_t[x_{t+1}] &= \alpha_{21}y_t + \alpha_{22}y_{t-1} + \beta_{21}x_t + \beta_{22}x_{t-1} + E_t[u_{2,t+1}] \\
&= \alpha_{21}y_t + \alpha_{22}y_{t-1} + \beta_{21}x_t + \beta_{22}x_{t-1}
\end{aligned}
$$

- Simple econometric methods (e.g., OLS) and inference tests can be used to identify the optimal number of lags and to estimate the coefficients.

Overview
D. Vector Autoregressions - pros and cons

- Pros:
  - Easy and fast
  - No exogenous variable to forecast
- Cons:
  - Need a large sample size, especially if one wants to predict more than 4 or 5 variables
  - Still a “Black Box” (no economic theory)

Overview
Structural econometric models

- Built using macroeconomic theory, estimated using econometric methods
- Example:
  - Economic theory tells us that exports depend on competitiveness and global demand
  - Using past observations, estimate short run and long run elasticities of exports with respect to:
    - global demand
    - competitiveness
  - Forecast global demand and competitiveness.
  - Use these forecasts and estimated elasticities to forecast exports.

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Overview
Structural models - Pros and Cons

Pro
- Can accompany figures with an economic "story"
- Allows for simulations – useful for policy analysis

Cons
- Require forecast of exogenous variables
- Can be very subjective

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Evaluation of Forecasts

- Forecast errors
- Forecast comparison
- Forecast pooling

Evaluation of Forecasts

Sources of forecast errors

Sources

a) Inappropriately formulated model

b) Inaccurately estimated model

c) Data generating process (DGP) has changed over time in unanticipated ways

Evaluation of Forecasts
Assessing forecast performance

Suppose you have data from time 0 up to time T.

In-sample analysis

- Estimate model using all available data up to time T, then compare the model's fitted values to the actual realizations

Out-of-sample analysis

- True out-of-sample. Estimate model until T, construct a forecast for T+1. Wait until T+1, record the forecast error and restimate model to make a forecast for T+2. Do this for a number of periods until there is sufficient forecast errors recorded to allow for an assessment o the model

- Pseudo (simulated) out-of-sample. Split the dataset into two period $(0,T_0)$, and (T_0+1, T).

Estimate model over $(0,T_0)$, then use model to forecast over (T_0+1,T)

Advantage: no need to wait

2 options: Dynamic forecast vs. Static Forecast. Dynamic: At time T_0+2, we use the forecast at time T_0+1 to predict T_0+2. Static: At time T_0+2, we use the actual realizations at time T_0+1 to forecast T_0+2

Evaluation of Forecasts
Forecast comparison

- Mean Absolute Error (MAE): mean of absolute values of forecast errors
- Mean Square Error (MSE): mean of squared forecast errors
- Root Mean Square Error (RMSE): square root of MSE
- Relative RMSE: ratio of the forecast RMSE to the RMSE of a benchmark forecast. A relative RMSE smaller than one indicates that the forecast is better than the benchmark. When the benchmark is a random walk forecast, the relative RMSE is called Theil coefficient

Evaluation of forecasts

Example: comparing one semester ahead OECD and random walk forecasts for French output gap

OECD forecast Root Mean Square Error =(0,734)$^{(1/2)}$=0,857 ...+1,636+0,543
+0,101+0,193+0,069+1,499)/8=0,626
Random Walk forecast Root Mean Square Error =1,05
Theil coefficient=0,857/1,058=0,810

	Actual figure	OECD Forecast	Forecast Error	Absolute Error	Random Walk forecast	Forecast Error	Absolute Error
1995	-2,572	-2,8	0,228	0,228	-1,595	-0,977	0,977
1996	-3,342	-2,6	-0,742	0,742	-2,572	-0,77	0,77
1997	-3,936	-2,3	-1,636	1,636	-3,342	-0,594	0,594
1998	-1,943	-1,4	-0,543	0,543	-3,936	1,993	1,993
1999	-0,499	-0,6	0,101	0,101	-1,943	1,444	1,444
2000	0,407	0,6	-0,193	0,193	-0,499	0,906	0,906
2001	0,669	0,6	0,069	0,069	0,407	0,262	0,262
2002	0,999	-0,5	1,499	1,499	0,669	0,33	0,33

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Evaluation of Forecasts

Forecast comparison: Example

- OECD forecast Mean Absolute Error = (0,228+0,742+1,636+0,543 +0,101+0,193+0,069+1,499)/8=0,626
- OECD forecast Mean Square Error =(0,052+0,551+2,676+0,295 +0,010+0,037+0,005+2,247)/8=0,734
- OECD forecast Root Mean Square Error =(0,734)$^{(1/2)}$=0,857
- Random Walk forecast Root Mean Square Error =1,058
- Theil coefficient=0,857/1,058=0,810

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5. Evaluation of Forecasts
Forecast Pooling

- Rather than selecting one out of many alternative forecasts for the same variable, we could combine them. The combined (pooled) forecast is

$$E_t[y_{t+1}^P] = \alpha_1 E_t[y_{t+1}^1] + \alpha_2 E_t[y_{t+1}^2] + \ldots + \alpha_N E_t[y_{t+1}^N]$$

- For example, we could pool forecasts based on the random walk model (the winner of a forecasting competition) and the structural model with economic story

- There are many methods to select the combination weights $\alpha_1,\ldots\alpha_N$. In practice, equal weights, $\alpha_1 = 1/N,\ldots\ \alpha_N = 1/N$, works well, possibly after dropping the worst forecasts from the set under analysis.

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Focus of macroeconomic part of the course

- Focus is on a suite of time series models that can be used as to forecast key variables of interest for central banks and finance ministries.
- Elements we will cover
  - Understanding time-series properties
  - Model design and economic theory
  - Model evaluation and diagnostic testing
  - Forecasting using your preferred model
- Monday – Thursday: Lectures followed by a practical session
- Friday: Case study and presentation
- Macro part of the course provides a good basis to complete the online course in MFx: Macroeconomic Forecasting. Next offering will start September 14. Course lasts 7 weeks, and require 6-7 hours of study weekly.

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Case Study
Modeling and Forecasting US Saving Rate

Purpose: to develop and improve your modeling and forecasting techniques by analyzing a current issue with a real dataset.

Case study: Forecasting U.S. Saving Rate

Data: Eviews **MF.WF1** file, pagefile("USA_CY"))

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L4 – Cointegration: Single Equation Estimation, Error Correction Models, and Forecasting

Fazeer Rahim (IMF East AFRITAC)

Outline of the lecture

1. The concept of cointegration
2. Spurious regressions: regressing variables that are not cointegrated
3. Testing for cointegration
4. Estimating the cointegrating vector
5. Cointegration and Error-Correction Models

Part I. Cointegration and the concept of equilibrium relationship

The concept of cointegration

Some theories imply the existence of a long-run equilibrium relationship between economic variables:

- Friedman's permanent income hypothesis: there is a link between consumption and disposable income
- purchasing power parity: prices developments in two countries, and changes in nominal exchange rate are such that the real exchange rate is constant

In most cases, these theories explain the joint behavior of non-stationary variables over the long-run

Introduction

Cointegration is the framework for testing, and estimating:

- the equilibrium relationship between non-stationary variables
- the short term dynamic behavior of these variables around the long-run equilibrium path

The permanent income hypothesis

Consider Friedman's permanent income hypothesis:

$$C_t^* = \beta Y_t^P$$

- C_t^* is the desired or equilibrium level of consumption
- Y_t^P is permanent disposable income

At each point in time:

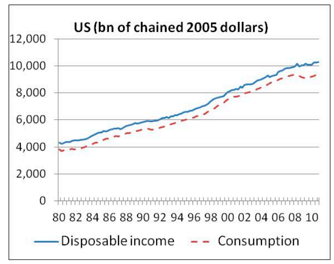
- *actual* consumption C_t may differ from C_t^*
- *actual* disposable income Y_t may differ from Y_t^P

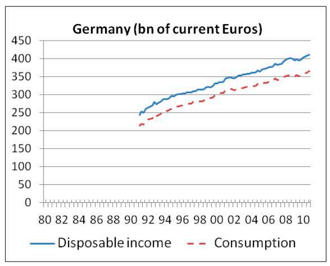
$$C_t = \beta Y_t + z_t$$

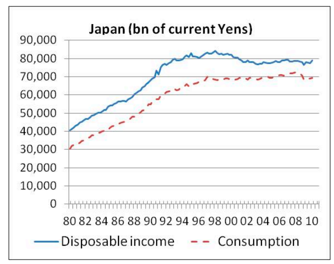
If the theory holds, z_t (transitory consumption) must be random and stationary.

The permanent income hypothesis

Disposable income and consumption for selected countries







Purchasing power parity

According purchasing power parity (PPP) :

$$E_t \frac{P_t^*}{P_t} = 1$$

- $P_t{}^*$ and P_t are the price levels in the foreign and domestic economy (generally, they are non-stationary)
- E_t is the nominal exchange rate, units of domestic currency per one unit of foreign currency (generally, non stationary)

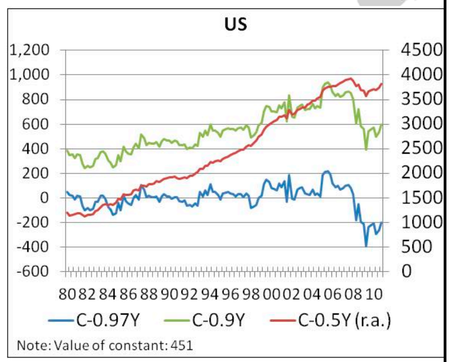
At each period, the actual exchange rate may differ from $\frac{P_t}{P_t^*}$ (taking logs)

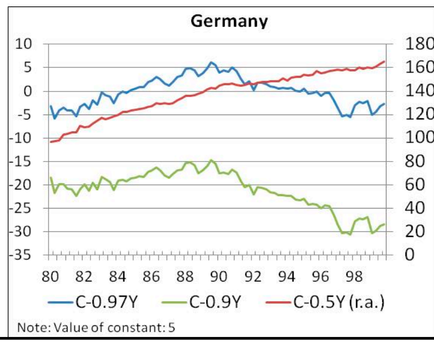
$$e_t + (p_t^* - p_t) = z_t$$

- As far as the deviation z_t are random and stationary, PPP holds

Cointegration is the exception not the norm

In general, *a generic* linear combination of non-stationary variables is non-stationary





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Cointegration defined

The variables Y_t and $X_{1,t},\ldots X_{k,t}$ are cointegrated if:

- Y_t , $X_{1,t},\ldots X_{k,t}$ are all I(1), and
- **there exists a vector of non zero coefficient [1,-β_1,... -β_k]** such that the linear combination

$$z_t = Y_t - \beta_1 X_{1,t} \ldots - \beta_k X_{k,t}$$

is a **stationary ARMA(p,q) process**.

It what follows we assume that there is only ONE co-integrating vector: that is, the variables $X_1,\ldots X_k$ are not cointegrated themselves, and they are not influenced by Y (no feedback).

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Cointegration and integration of same order

Another way in which Y_t and X_t are *not* cointegrated is if:

- the variables are integrated of different order (Y_t is I(p), while X_t is I(q))

For example, suppose that Y_t is I(1) and X_t is stationary:

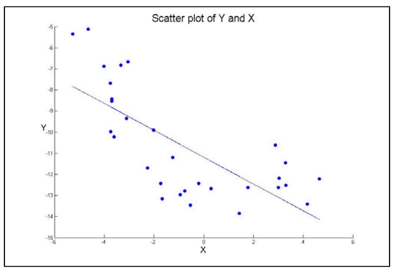
- How can you possibly explain the long-run growth of Y_t (the trend) with a variable that does not grow in the long-run (it actually tends to revert to a fixed constant)?

Part II. Spurious regressions

OLS on I(1) un-related variables

Consider regressing the following non-stationary variables:

$$y_t = y_{t-1} + \varepsilon_{y,t} \qquad \varepsilon_{y,t} \propto N(0,1)$$
$$x_t = x_{t-1} + \varepsilon_{x,t} \qquad \varepsilon_{x,t} \propto N(0,1)$$



$y_t = -11.181 - 0.637 x_t$

t-stat.: -5.5654

$R^2 : 0.52$

Refresher of OLS assumptions

Refresher on Ordinary Least Square assumptions $y_t = \alpha + \beta x_t + z$

$$E[z_t] = 0$$
$$Var[z_t] = \sigma^2$$
$$Cov[z_t z_{t-j}] = 0$$
$$Cov[z_t z_{t-j}] = 0$$

OLS estimates are unbiased and have the minimum variance compared to other estimators

Implication of above is that error term z_t is stationary

But when you regress two non-stationary variables which are not-cointegrated, the error term is not stationary

Similar reasoning for misspecified models

Consider the following non-stationary variables:

$$x_t = x_{t-1} + \varepsilon_{x,t} \qquad \varepsilon_{y,t} \sim N(0,1)$$

$$z_t = z_{t-1} + \varepsilon_{z,t} \qquad \varepsilon_{x,t} \sim N(0,1)$$

$$y_t = 1 + 2x_t + 3z_{t-1} + \varepsilon_{y,t} \qquad \varepsilon_{y,t} \sim N(0,1)$$

Consider estimating with OLS the following mis-specified model:

$$y_t = \alpha + \beta x_t + \varepsilon_t$$

By construction, y_t and x_t are not cointegrated. The model is mis-specified because an I(1) variable is missing.

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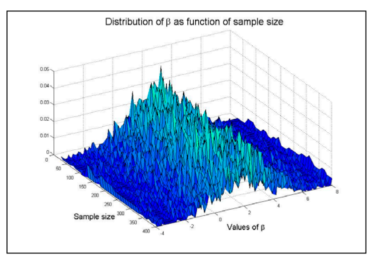
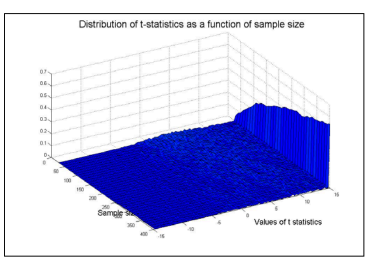
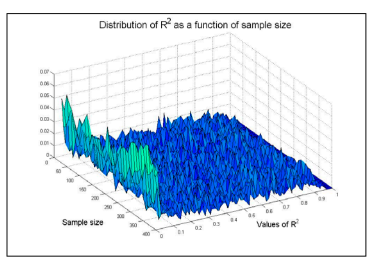
Monte Carlo results

The OLS estimates have the same problems as before: the distribution of $\hat{\beta}$ does not converge to the true value, even with large samples







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OLS and spurious regression

Conclusion: If you are modeling I(1) variables, it is crucial that the variables be cointegrated, otherwise results and tests do not make sense

- If the variable are not cointegrated, OLS regressions will provide spurious results
- Notice, if the variables are cointegrated, OLS estimates the cointegrating vector very well

Part III. Testing for cointegration

The Engle and Granger 2-Step Approach

Step 1. estimating the cointegration regression by OLS, and obtain the residual z_t

Step 2. Apply ADF test on z_t

Note: The null hypothesis (H_0) of the ADF test is the residual zt has a unit root. So, for cointegration, we need to have rejected the null hypothesis.

Summary

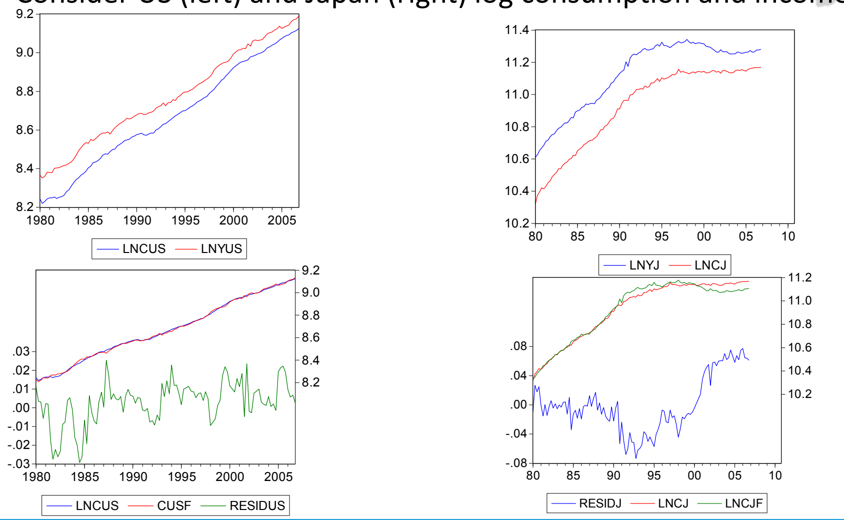
If the estimated residuals have a unit root, the model is NOT cointegrated.

If we don’t have co-integration:

- we either augment the long-run model by including additional I(1) variables
- or we simply reject our long-run theory/hypothesis, e.g. PPP.

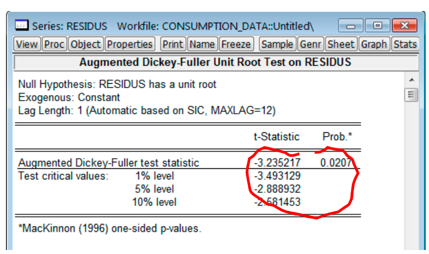
Example: US and Japan consumption

Consider US (left) and Japan (right) log consumption and income



Example: US and Japan consumption and income

US



Series: RESIDUS Workfile: CONSUMPTION_DATA::Untitled\

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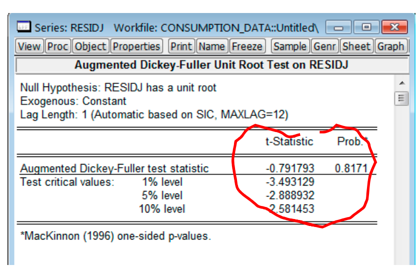
Augmented Dickey-Fuller Unit Root Test on RESIDUS

Null Hypothesis: RESIDUS has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic based on SIC, MAXLAG=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.235217	0.0207
Test critical values:	1% level	-3.493129	
	5% level	-2.888932	
	10% level	-2.581453	

*MacKinnon (1996) one-sided p-values.

Japan



Series: RESIDJ Workfile: CONSUMPTION_DATA::Untitled\

View Proc Object Properties Print Name Freeze Sample Genr Sheet Graph

Augmented Dickey-Fuller Unit Root Test on RESIDJ

Null Hypothesis: RESIDJ has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic based on SIC, MAXLAG=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.791793	0.8171
Test critical values:	1% level	-3.493129	
	5% level	-2.888932	
	10% level	-2.581453	

*MacKinnon (1996) one-sided p-values.

We cannot reject the presence of unit root in the residuals

Part IV. Estimating the Cointegrating Vector

Estimating the long-run relationship

If Y, X_1,…X_k are cointegrated, OLS provides an estimate of the cointegrating vector.

Under cointegration, OLS is "super consistent":

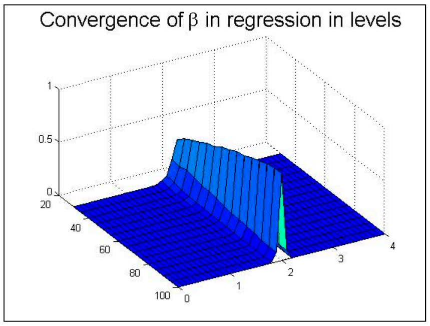
- estimates of the parameters approach the true values very quickly as the number of observations increases
- the estimated parameters are consistent even if any of X_1,…X_k is correlated with the error term (no asymptotic simultaneity bias; however, the bias may be substantial in small samples)
- R^2 approaches one as the number of observations increases

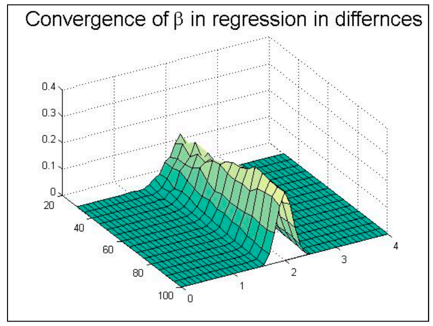
However, the OLS estimator can be badly biased in small sample due to correlations

Super consistency

Suppose that Y and X are cointegrated; estimate:

$$y_t = \alpha + \beta x_t + z_t \qquad \Delta y_t = \beta \Delta x_t + \Delta z_t$$





Inference in cointegrated systems

Now suppose we want to do some standard hypothesis tests using a cointegrated regression, for example: "is $\beta = 1$?"

While OLS produces acceptable parameter estimates, it does not, in general, yield valid standard errors (and hence t-statistics):

- residuals z_t, can be serially correlated and correlated with the innovations in the explanatory variables $\varepsilon_{x,t}$
- the correlations of z_t with $\varepsilon_{x,t}$ can induce substantial bias in small samples

FM and CCR approach

If the residuals of the co-integrating equation (z_t) and the changes in the regressors ($\varepsilon_{x,t}$) are correlated:

- the Fully Modified Least Squares (Phillips and Hansen) "FM"
- the Canonical Cointegrating Regressions (Park) "CCR"

obtain asymptotically unbiased and fully efficient estimates of the parameters.

Conclusion: they produce estimates that can be used for hypothesis testing

Broadly speaking, both methods (both available in recent versions of EViews):

- estimate the long run correlation between z_t and $\varepsilon_{x,t}$
- adjust y_t using these estimated long run correlation
- use the adjusted y_t to estimate parameters

Part V. Cointegration and Error Correction Models

Granger representation theorem

Engle and Granger (1987) show that co-integration implies the existence of an "error correction model" (ECM) of the form:

$$\Delta y_t = c + \alpha(y_{t-1} - \beta x_{t-1}) + \sum_{j=1}^{p} \phi_j \Delta y_{t-j} + \sum_{j=1}^{p} \gamma_j \Delta x_{t-j} + \varepsilon_t$$

- β expresses the **long-run equilibrium relationship**
- α expresses the **speed of adjustment**, or how strongly the past disequilibrium affects changes in y
- p is the order of the AR process in y and x
- ϕ and γ are the parameters of the "own" dynamics of y and x (not reflecting disequilibrium conditions)

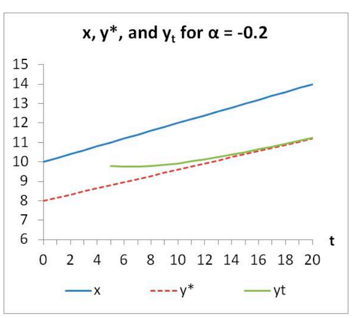
Idea: Cointegrated variables both wander stochastically, but stay near each other. They adjust to each other's locations through a process of error correction. The ECM shows this adjustment process.

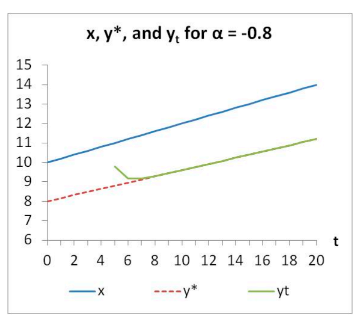
The adjustment towards equilibrium

The parameter α (also called the factor loading) indicates the speed of adjustment towards equilibrium. Consider

$$\Delta y_t = 0.16 + \alpha(y_{t-1} - 0.8x_{t-1}) + \varepsilon_t$$

Suppose that x grows linearly, the system is in equilibrium, but it receives a shock of 1 at t = 5.





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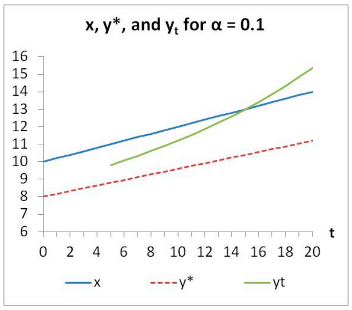
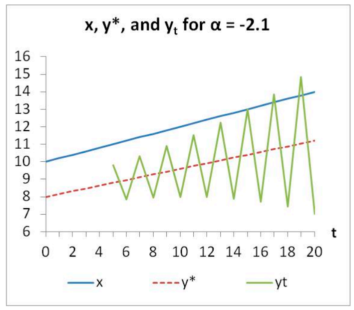
Constraints on α

In order for the ECM to be well-defined (and for cointegration)

$$-2 < \alpha < 0$$

- α < 0 so that y can go back to equilibrium after a deviation (a positive deviation requires less of an increase in y)
- α > - 2 otherwise there is "overshooting" with oscillations





Estimation of ECM

STEP 1: Establish cointegration

STEP 2: If cointegration is established, estimate ECM

Further checks:

- Does the coefficient on the adjustment process imply convergence to equilibrium?
- Is any non-stationary variable missing? (check this by looking at the residuals of the ECM…are they white noise?)
- Are the residuals of the ECM non-stationary? If so, there may be structural break in the sample. If that is the case, add dummies.

Example of ECM: Deficits*

- Example: Interest Rates and Deficits
- Do large Federal budget deficits drive up long-term interest rates?
- y: 10 year treasury bond rate
- x1: the 1 year rate, x2: inflation, x3: the real deficit per capita, x4: the change in real per capita income.

Source: Econometrics A Modern Introduction by M. Murray, 2006

Example of ECM: Deficits

- Augmented Dickey–Fuller tests suggest that the interest rates, inflation, deficit per capita, and level of income per capita are all integrated I(1).
- OLS regression (on the right) is run.
- Augmented Dickey-Fuller test (below) is done on the residual. Result: we reject the null hypothesis of unit root in the residual. Thus, there is cointegration

Dependent Variable: FYGT10
Method: Least Squares
Sample(adjusted): 1953 1998
Included observations: 46 after adjusting endpoints
Newey–West HAC Standard Errors & Covariance (lag truncation = 3)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.264513	0.119566	10.57584	0.0000
FYGT1	0.826709	0.035165	23.50950	0.0000
INFL	−0.055646	0.032913	−1.690684	0.0985
USDEF	0.006687	0.000389	17.18137	0.0000
DY	8.754592	135.1886	0.064758	0.9487
R-squared	0.984579	Mean dependent var		6.750272
Adjusted R-squared	0.983074	S.D. dependent var		2.822053
S.E. of regression	0.367148	Akaike info criterion		0.936220
Sum squared resid	5.526714	Schwarz criterion		1.134986
Log likelihood	−16.53307	F-statistic		654.4105
Durbin–Watson stat	1.707091	Prob(F-statistic)		0.000000

Augmented Dickey–Fuller Test Equation
Dependent Variable: D(RESID01)
Method: Least Squares
Sample(adjusted): 1954 1998
Included observations: 45 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
RESID01(−1)	−0.859757	0.150067	−5.729148	0.0000
C	0.005599	0.052582	0.106474	0.9157

Forecasting

Assume that:

- the ECM model is correctly specified
- the speed of adjustment and equilibrium relationship will hold in the future

We can forecast the impact of a shock to one variable (e.g. one-off fall in permanent income) on the other dependent variable, distinguishing between the adjustment to the long-run relationship and the short-run impact (via the autoregressive components).

Conclusion

- Cointegration is the framework for testing, and estimating the equilibrium relationship between non-stationary variables and the short term dynamic behavior of these variables around the long-run equilibrium path
- Only if variables are cointegrated one can estimate the long-run relationship and the parameters of the ECM representation
- The parameter β captures the long-run relationship; $y = \beta x$ only holds in the long-run
- The parameter α captures the short-run dynamic around the equilibrium and must be negative to have cointegration

Thank you

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Workshop on Macro-Fiscal Forecasting
Arusha, August 8 – 18, 2016

L5 – Vector Autoregressions, Structural VAR Models, Impulse Response Function

Fazeer Rahim (IMF East AFRITAC)

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Multivariate Time-Series Models

- Definition - VAR
- Stationarity
- Choosing the lag length
- Forecasting using VARs
- Granger causality test
- The impulse response function
- Structural VAR

2

Definition - Vector Autoregressions (VAR)

Definition - VAR

What do to when we are not confident that a variable is actually exogenous, a natural extension is to treat each variable symmetrically – *i.e.*, endogenously.

VAR model is an extension of ***univariate*** autoregression (**AR**) model to multivariate (**Vector**) time series data.

VAR model is a multi-equation system where all the variables are treated as endogenous., i.e. there is one equation for each variable as the dependent variable.

VAR allows study of **joint** **dynamic and contemporaneous** relations between variables (autoregressive).

Do we really need VAR?

- Good at capturing co-movements of multiple time series, i.e. allows analysis of system response to different shocks/impacts.
- Do not need to specify which variables are endogenous or exogenous - all are endogenous!
- Allows the value of a variable to depend on more than just its own lags or combinations of white noise terms, so more general than ARMA modelling.
- Provided that there are no contemporaneous terms on the right hand side of the equations, can simply use OLS separately on each equation.
- VAR forecasts are often better than "traditional structural" models.
- Note: In VAR, variables must be of the **same frequency!**

What do we report in VAR?

- In VAR, we are not interested in coefficients.
- 3 main components:
    1. Granger-causality tests – does X cause movements in Y?
    2. Impulse response (IRFs) – innovations/shocks
    3. Forecasts of the variables
- These are more **informative** to understanding the **relationships** among variables than the VAR regression coefficients or the R^2 statistics.

Note: When the variables of a VAR are cointegrated, we use a vector error-correction model (**VECM**) – more on this tomorrow's lecture.

A VAR(1) with two variables

Consider a one-variable AR(1):

$$y_t = a + b_1 y_{t-1} + \varepsilon_t$$

A VAR(1) for two variables

In matrix form: $$\begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}$$

Or as a set of equations:

$$y_{1,t} = c_1 + b_{11} \cdot y_{1,t-1} + b_{12} \cdot y_{2,t-1} + \varepsilon_{1,t}$$
$$y_{2,t} = c_2 + b_{21} \cdot y_{1,t-1} + b_{22} \cdot y_{2,t-1} + \varepsilon_{2,t}$$

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Extension to many variables and lags

Let y_t be a vector with the value of *n* variables at time *t*:

$$\mathbf{y_t} = \begin{bmatrix} y_{1,t} \\ y_{2,t} \\ \dots \\ y_{n,t} \end{bmatrix} \qquad \text{for example: } \mathbf{y_t} = \begin{bmatrix} M_t \\ P_t \\ I_t \\ GDP_t \end{bmatrix}$$

A *p*-order *vector autoregressive process* (VAR) generalizes a one-variable AR(p) process to *n* variables

$$\mathbf{y_t} = \mathbf{c} + \mathbf{B_1 y_{t-1}} + \mathbf{B_2 y_{t-2}} + \dots + \mathbf{B_p y_{t-p}} + \boldsymbol{\varepsilon}_\mathbf{t}$$

$\mathbf{c}$ = (n × 1) vector of constants

$\mathbf{B_j}$ = (n × n) matrix of coefficients

$\boldsymbol{\varepsilon}_\mathbf{t}$ = (n × 1) vector of white noise innovations

$$E[\boldsymbol{\varepsilon}_\mathbf{t}] = 0$$

$$E[\boldsymbol{\varepsilon}_\mathbf{t}\boldsymbol{\varepsilon}_\boldsymbol{\tau}'] = \begin{cases} \Omega \text{ if } t = \tau \\ 0 \text{ otherwise} \end{cases}$$

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Estimation of VAR

$$\mathbf{y}_t = \mathbf{c} + \mathbf{B}_1\mathbf{y}_{t-1} + \mathbf{B}_2\mathbf{y}_{t\text{-}2} + \ldots + \mathbf{B}_p\mathbf{y}_{t\text{-}p} + \boldsymbol{\varepsilon}_t$$

Since no current values of Y_t appear on the right hand side, the above VAR model can easily be estimated by OLS and able to produce asymptotically desirable estimators.

Stationarity

10

Stationarity of a VAR: Definition

A p-th order VAR is said to be covariance-stationary if:

- the expected value of $\mathbf{y_t}$ does not depend on time

$$E[\mathbf{y}_\mathbf{t}] = E[\mathbf{y}_{\mathbf{t+j}}] = \boldsymbol{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \cdots \\ \mu_n \end{bmatrix}$$

- The covariance matrix of $\mathbf{y_t}$ and $\mathbf{y_{t-j}}$ depends on the time lapsed j and not on the reference period t

$$E[(\mathbf{y}_\mathbf{t} - \boldsymbol{\mu})(\mathbf{y}_{\mathbf{t-j}} - \boldsymbol{\mu})'] = E[(\mathbf{y}_\mathbf{s} - \boldsymbol{\mu})(\mathbf{y}_{\mathbf{s-j}} - \boldsymbol{\mu})'] = \boldsymbol{\Gamma}_{\mathbf{-j}}$$

$$\boldsymbol{\Gamma}_\mathbf{j}' = \boldsymbol{\Gamma}_{\mathbf{-j}}$$

If a VAR process is covariance stationary then all of its n components are stationary.

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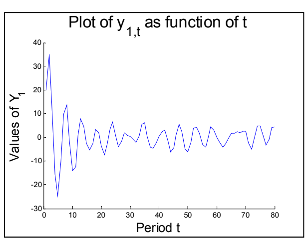
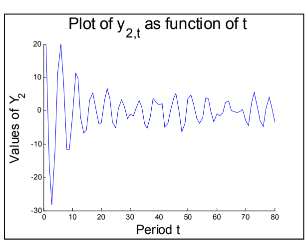
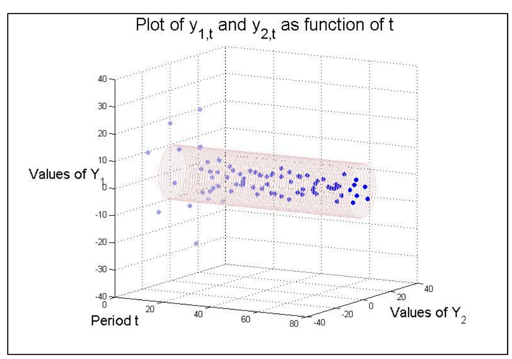
Example of a stationarity VAR

In a stationary VAR variables eventually stabilize within a band







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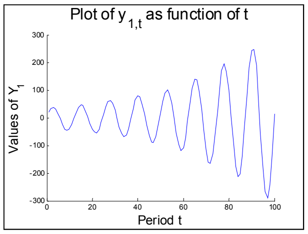
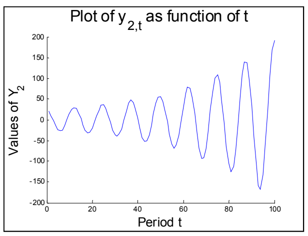
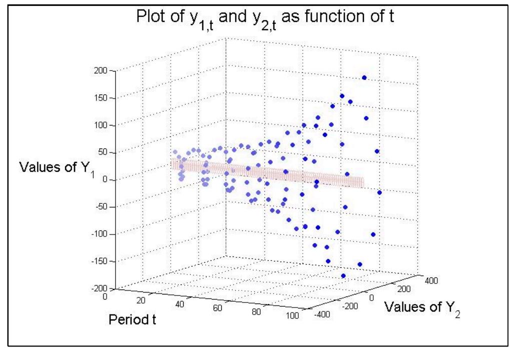
Example of a non-stationary VAR

If a VAR is non-stationary, variables diverge

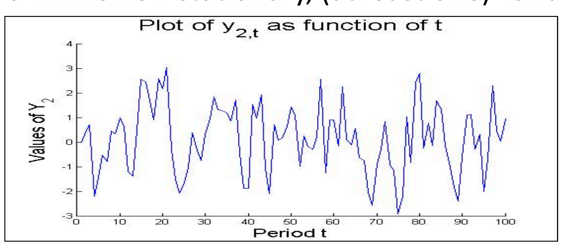


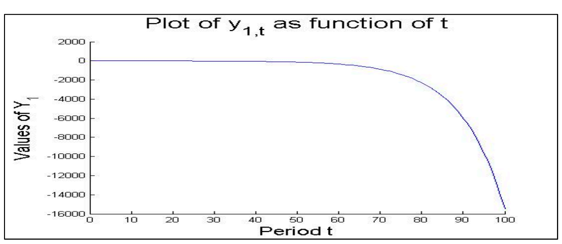




Example of a non-stationary VAR

If a VAR is non-stationary, (at least one) variable diverges





Stationarity of a VAR: Definition

If a VAR is stationary, then:

$$\mathbf{\mu} = \mathbf{c} + \mathbf{B}_1\mathbf{\mu} + \mathbf{B}_2\mathbf{\mu} + \ldots + \mathbf{B}_p\mathbf{\mu}$$

Substituting into the VAR equation:

$$(\mathbf{y}_t - \mathbf{\mu}) = \mathbf{c} + \mathbf{B}_1(\mathbf{y}_{t-1} - \mathbf{\mu}) + \mathbf{B}_2(\mathbf{y}_{t-2} - \mathbf{\mu}) + \ldots + \mathbf{B}_p(\mathbf{y}_{t-p} - \mathbf{\mu}) + \mathbf{\varepsilon}_t$$

The vector $\mathbf{\mu}$ contains the "equilibrium" values of each variable:

- Because $E[\varepsilon] = 0$ and these innovations are independent across time, their effect cancels out over time
- Deviations of each variable from equilibrium reduce over time; each variable eventually converges to its expected value.

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Consequences of stationarity for estimation

If a VAR is stationary, one can estimate the parameters of the process:

- the rows of the **B** matrices by simple OLS, line by line
- the covariance matrix $\boldsymbol{\Omega}$ with the OLS residuals
- the autocovariance matrices $\boldsymbol{\Gamma}$ by estimates of **B** matrices and the matrix $\boldsymbol{\Omega}$

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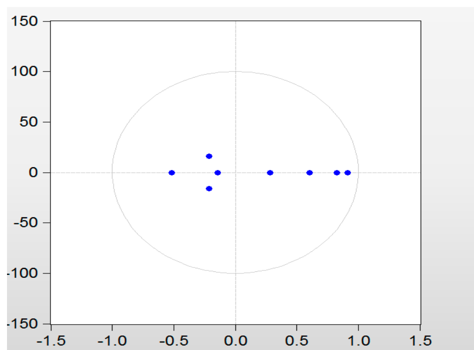
Conditions for Stationarity

The conditions for a VAR to be stationary are similar to the conditions for a univariate AR

All the roots are to lie within the unit circle



Moving-Average Representation of a VAR

If a VAR is stationary, the $\mathbf{y}_t$ vector can be expressed as a sum of all of the past vector white noise shocks

$$\mathbf{y}_t = \boldsymbol{\mu} + \sum_{i=0}^{\infty} \boldsymbol{\Psi}_i \boldsymbol{\varepsilon}_{t-i}$$

where $\boldsymbol{\Psi}_i$ is a (n x n) matrix of coefficients, and $\boldsymbol{\Psi}_0$ is the identity matrix.

In a first order AR process, in an AR protability in univariate:

Stability

Choosing the lag length

How Many Lags Needed for the VAR?

What is the most appropriate number of lags?

Trade-off:

- If p is short, model may be misspecified. Consequence: residuals will be autocorrelated over time, hence breaking OLS assumption.

- If p is long, too many degrees of freedom will be lost, i.e. every extra lag add n-squared more coefficients to estimate. Need a lot of data points.
  - Little data and long p leads to overfitting: As the VAR tries to estimate many coefficients with little data, the coefficients themselves are poorly estimated. However, the model appears to fit the in sample data very well.But out of sample forecasts are actually very bad.

How Many Lags Needed for the VAR?

As for univariate models, one can use a multi-dimensional version of the AIC and BIC criterion to find the optimal value of the number of *common* lags *p*:

- Schwartz Bayesian Criterion (SBC)

$$SBC = T\ln(\det(\mathbf{\Omega})) + N\ln(T)$$

- Akaike Information Criterion (AIC)

$$AIC = T\ln(\det(\mathbf{\Omega})) + 2N$$

where N is the total number of parameters to estimate in all equations

Forecasting using VARs

Forecasting using the estimated VAR

Like univariate stationary AR processes, the estimated VAR process can also be used to make forecasts.

Specifically, let

$$\mathbf{x}_{\text{t-1}} = (\mathbf{y}_{\text{t-1}}, \mathbf{y}_{\text{t-2}}, \ldots, \mathbf{y}_{\text{t-T}})$$

be a matrix containing all information available up to time *t*, before realizations of $\boldsymbol{\varepsilon}_{\text{t}}$ are known. Then:

$$E[\mathbf{y}_{\text{t}} \mid \mathbf{x}_{\text{t-1}}] = \hat{\mathbf{c}} + \hat{\mathbf{B}}_1 \mathbf{y}_{\text{t}-1} + \hat{\mathbf{B}}_2 \mathbf{y}_{\text{t-2}} + \ldots + \hat{\mathbf{B}}_{\text{p}} \mathbf{y}_{\text{t-p}}$$

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Forecasting using the estimated VAR

The forecast error is the sum of $\boldsymbol{\varepsilon}_{\text{t}}$, the unexpected innovation of $\mathbf{y}_{\text{t}}$, and the coefficient estimation error

$$\mathbf{y}_{\text{t}} - E[\mathbf{y}_{\text{t}} \mid \mathbf{x}_{\text{t-1}}] = \boldsymbol{\varepsilon}_{\text{t}} + \nu(\mathbf{x}_{\text{t-1}})$$

If the estimator for the coefficients is consistent and estimates are based on many data observations, the coefficient estimation error tends to be small, and

$$\mathbf{y}_{\text{t}} - E[\mathbf{y}_{\text{t}} \mid \mathbf{x}_{\text{t-1}}] \cong \boldsymbol{\varepsilon}_{\text{t}}$$

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Forecasting using the estimated VAR

Iterating one period forward:

$$E[\mathbf{y}_{t+1} \mid \mathbf{x}_{t-1}] = \hat{\mathbf{c}} + \hat{\mathbf{B}}_1 E[\mathbf{y}_t \mid \mathbf{x}_{t-1}] + \hat{\mathbf{B}}_2 \mathbf{y}_{t-1} + \ldots + \hat{\mathbf{B}}_p \mathbf{y}_{t-p+1}$$

$$\mathbf{y}_{t+1} - E[\mathbf{y}_{t+1} \mid \mathbf{x}_{t-1}] \cong \hat{\mathbf{B}}_1 (\mathbf{y}_t - E[\mathbf{y}_t \mid \mathbf{x}_{t-1}]) + \boldsymbol{\varepsilon}_{t+1} \cong \boldsymbol{\varepsilon}_{t+1} + \hat{\mathbf{B}}_1 \boldsymbol{\varepsilon}_t$$

Iterating *j* period forward:

$$E[\mathbf{y}_{t+j} \mid \mathbf{x}_{t-1}] = \hat{\mathbf{c}} + \hat{\mathbf{B}}_1 E[\mathbf{y}_{t+j-1} \mid \mathbf{x}_{t-1}] + \hat{\mathbf{B}}_2 E[\mathbf{y}_{t+j-2} \mid \mathbf{x}_{t-1}] + \ldots + \hat{\mathbf{B}}_p \mathbf{y}_{t-p+j}$$

$$\mathbf{y}_{t+j} - E[\mathbf{y}_{t+j} \mid \mathbf{x}_{t-1}] \cong \boldsymbol{\varepsilon}_{t+j} + \hat{\Psi}_1 \boldsymbol{\varepsilon}_{t+j-1} + \hat{\Psi}_2 \boldsymbol{\varepsilon}_{t+j-2} + \ldots + \hat{\Psi}_j \boldsymbol{\varepsilon}_t$$

Granger Causality Test

Granger causality

X "Granger causes" Y if past values of X can help explain Y

Granger causality is predictive causality

Granger causality

It is likely that, when a VAR includes many lags of variables, it will be **difficult** to see which sets of variables have *significant effects on each dependent variable* and which do not. Consider the following VAR(2)

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} c_y \\ c_z \end{pmatrix} + \begin{pmatrix} b_{11}^{(1)} & b_{12}^{(1)} \\ b_{21}^{(1)} & b_{22}^{(1)} \end{pmatrix} \begin{pmatrix} y_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} b_{11}^{(2)} & b_{12}^{(2)} \\ b_{21}^{(2)} & b_{22}^{(2)} \end{pmatrix} \begin{pmatrix} y_{t-2} \\ x_{t-2} \end{pmatrix} + \begin{pmatrix} \varepsilon_{y,t} \\ \varepsilon_{x,t} \end{pmatrix}$$

Consider a *F-test* on the hypothesis (Granger causality):

$$H_0 : b_{12}^{(1)} = b_{12}^{(2)} = 0$$

If the H_0 is rejected, then we reject the null hypothesis that "*x* does not *Granger cause y*":

Granger causality

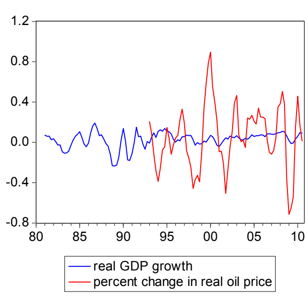
Rejecting the Granger causality test (➡ *x* "Granger causes" *y*):

- If economic theory or our knowledge, it suggests that x causes y, use test result to confirm the theory
- If not, it simply states that *x* improves the forecast of *y*

  Example: an increase in orange prices today does not cause a poor harvest. But we may use our observation of this event to infer about a poor harvest.

VAR and causality: example 1

Consider a 8-order VAR with quarterly data on:
- y = Peru's GDP Q-o-Q growth (at 1994 prices)
- x = Q-o-Q increase in price of crude oil (at 1994 prices)



Dependent variable: LNGDP-LNGDP(-4)

Excluded	Chi-sq	df	Prob.
LNOILP-LNOILP(-4)	24.65763	8	0.0018
All	24.65763	8	0.0018

Dependent variable: LNOILP-LNOILP(-4)

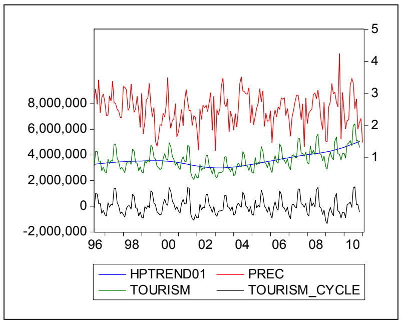
Excluded	Chi-sq	df	Prob.
LNGDP-LNGDP(-4)	10.76990	8	0.2151
All	10.76990	8	0.2151

Changes in oil prices Granger cause Peru's growth: we can interpret this as "Peru's GDP is affected by the real cost of oil"

VAR and causality: example 2

Consider a 8-order VAR with monthly data on:

- y = average precipitation in the US (in inches)
- x = number of tourist arrival from abroad (de-trended)



Dependent variable: PREC

Excluded	Chi-sq	df	Prob.
TOURISM_...	33.58978	8	0.0000
All	33.58978	8	0.0000

Dependent variable: TOURISM_CYCLE

Excluded	Chi-sq	df	Prob.
PREC	33.92570	8	0.0000
All	33.92570	8	0.0000

R2 = 0.32 (against 0.15 if tourism is excluded): tourist arrival might help predict the weather, but it does not cause it

The Impulse Response Function (IRF)

The impulse response function

- VAR models are often difficult to interpret: one solution is to construct the impulse responses and variance decompositions.

An impulse response function *traces the responsiveness of a dependent variable in the VAR to a shock to one variables, keeping other shocks to zero*.

In a VAR, one can identify a set of NxN IRFs, corresponding to the response of each variable from a shock in each variable. Usually we use 1 standard deviation shock.

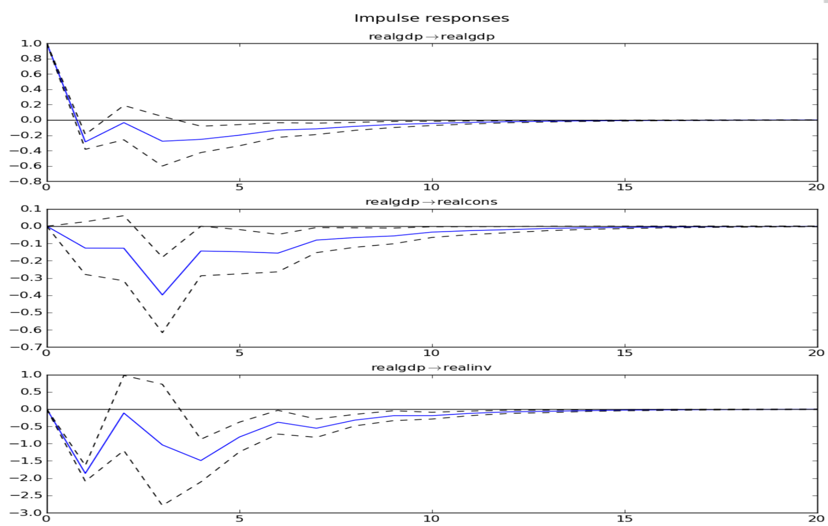
- Consider for example a simple bivariate VAR(1):

$$y_{1t} = \beta_{10} + \beta_{11} y_{1t-1} + \alpha_{11} y_{2t-1} + u_{1t}$$
$$y_{2t} = \beta_{20} + \beta_{21} y_{2t-1} + \alpha_{21} y_{1t-1} + u_{2t}$$

- A change in u_{1t} will immediately change y_1. It will change y_2 and also y_1 during the next period, etc

- We can examine **how long** and to **what degree** a shock to a given equation has on all of the variables in the system

Example: IRF for shock to real GDP



Constructing the impulse response function

Construct the IRF by simulating the effect on $y_{1,t}$, of a one-unit innovation $\varepsilon_{1,1}$

$$\mathbf{y_t} = \begin{pmatrix} 0.9 & 0.2 \\ -0.3 & 0.2 \end{pmatrix} \mathbf{y_{t-1}} + \boldsymbol{\varepsilon}_\mathbf{t} \qquad \mathbf{y}_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \ \boldsymbol{\varepsilon}_\mathbf{1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ \boldsymbol{\varepsilon}_\mathbf{1+j} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

How would the IRF look like if we had?

$$\mathbf{y_t} = \begin{pmatrix} 0.9 & 0.2 \\ 0 & 0.2 \end{pmatrix} \mathbf{y_{t-1}} + \boldsymbol{\varepsilon}_\mathbf{t}$$

Structural VAR

Reduced Form vs Structural VAR

- A *reduced-form VAR* expresses each variable as a linear function of its own past values and the past values of all other variables being considered and a serially uncorrelated error term. The error terms are viewed as "surprises"—movements in the variables after taking its past into account.
- A *structural VAR* (SVAR)uses **economic theory** to sort out contemporaneous links among the variables. SVARs require "identifying assumptions" that establish causal links among variables.

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Structural VAR Model

- So far, we have assumed the **VAR is autoregressive**
- But what if the equations had a **contemporaneous** feedback term?

$$y_{1t} = \alpha_{10} + \alpha_{12t} y_{2t} + \beta_{11t} y_{1(t-1)} + \beta_{12} y_{2(t-1)} + e_{1t}$$

$$y_{2t} = \alpha_{20} + \alpha_{21t} y_{1t} + \beta_{21t} y_{2(t-1)} + \beta_{22} y_{2(t-1)} + e_{2t}$$

- This is the **structural** VAR :

$$\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} \alpha_{10} \\ \alpha_{20} \end{pmatrix} + \begin{pmatrix} \alpha_{12} & 0 \\ 0 & \alpha_{21} \end{pmatrix} \begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} + \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} \begin{pmatrix} y_{1t-1} \\ y_{2t-1} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix}$$

- In more general form with *m* variables and *p* lags:

$$\mathbf{y_t} = \mathbf{c} + \mathbf{B}_0 \mathbf{y}_t + \mathbf{B}_1 \mathbf{y}_{t-1} + \mathbf{B}_2 \mathbf{y}_{t-2} + \ldots + \mathbf{B_p y_{t-p}} + e_{\mathbf{t}}$$

This VAR **can't be estimated** by OLS. Need **reduced** VAR. Why??

Reduced VAR Model

- To obtain **reduced** form VAR from **structural** VAR, take the contemporaneous variables to the LHS:

$$\begin{bmatrix} 1 & -\alpha_{12} \\ -\alpha_{21} & 1 \end{bmatrix}\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} \alpha_{10} \\ \alpha_{20} \end{pmatrix} + \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix}\begin{pmatrix} y_{1t-1} \\ y_{2t-1} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix}$$

- Multiply inverse of this term on the RHS: $\longrightarrow \begin{pmatrix} 1 & -\alpha_{12} \\ -\alpha_{21} & 1 \end{pmatrix}^{-1}$

$$\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} b_{10} \\ b_{20} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}\begin{pmatrix} y_{1t-1} \\ y_{2t-1} \end{pmatrix} + \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix}$$

More generally;

$$[I - B_0]\mathbf{y_t} = \mathbf{c} + \mathbf{B_1 y_{t-1}} + \mathbf{B_2 y_{t-2}} + \ldots + \mathbf{B_p y_{t-p}} + e_{\mathbf{t}}$$

Thus **reduced VAR** which can be **estimated by OLS** is:

$$\mathbf{y_t} = [I - B_0]^{-1}(\mathbf{c} + \mathbf{B_1 y_{t-1}} + \mathbf{B_2 y_{t-2}} + \ldots + \mathbf{B_p y_{t-p}} + e_{\mathbf{t}})$$

Identification in SVAR Model

- Remember that we started with a structural VAR model, and jumped into the reduced form or standard VAR for estimation purposes
- Is it possible to recover the parameters in the structural VAR from the estimated parameters in the reduced VAR?
- There are **8 parameters** in the bivariate ***structural*** VAR(1) and only **7 estimated parameters** in the ***reduced*** VAR(1)
- The VAR is **underidentified**
- If one parameter in the structural VAR is **restricted** the reduced VAR is **exactly identified**, therefore, able to *recover* parameters
- Cholesky decomposition: rank the variables to that
  - First variable responds to the contemporaneous shocks of all variables
  - Second variables does not respond to shocks of 1st variables, but responds to all other shocks
  - …
  - Last variables does not reponds to any shock in the other variables

Identification in SVAR Model

α_{21}= 0 implies

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} 1 & -\alpha_{12} \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \beta_{10} \\ \beta_{20} \end{bmatrix} + \begin{bmatrix} 1 & -\alpha_{12} \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} y_{1t-1} \\ y_{2t-1} \end{bmatrix} + \begin{bmatrix} 1 & -\alpha_{12} \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} \phi_{10} \\ \phi_{20} \end{bmatrix} + \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} y_{1t-1} \\ y_{2t-1} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}$$

The parameters of the structural VAR can now be identified from the following 9 equations. Therefore, **exactly identified!**

$$\phi_{10} = \beta_{10} - \alpha_{12}\beta_{20} \quad \phi_{20} = \beta_{20} \quad \mathrm{var}(e_1) = \sigma_1^2 + \alpha_{12}^2\sigma_2^2$$

$$\phi_{11} = \beta_{11} - \alpha_{12}\beta_{21} \quad \phi_{21} = \beta_{21} \quad \mathrm{var}(e_2) = \sigma_2^2$$

$$\phi_{12} = \beta_{12} - \alpha_{12}\beta_{22} \quad \phi_{22} = \beta_{22} \quad \mathrm{cov}(e_1, e_2) = -\alpha_{12}\sigma_2^2$$

Identification in SVAR Model

- Note both structural shocks can now be identified from the residuals of the standard VAR
- **α_{21}= 0** implies $\mathbf{y_1}$ does not have a contemporaneous effect on $\mathbf{y_2}$
- This restriction manifests itself such that both u_1 and u_2 affect $\mathbf{y_1}$ contemporaneously but only u_2 affects y_2 contemporaneously
- The residuals of e_{2t} are due to pure shocks to $\mathbf{y_2}$
- Decomposing the residuals of the reduced VAR in this **triangular** fashion is called the **Choleski decomposition**
- There are other methods used to identify models, like Blanchard and Quah (1989) decomposition, etc

Conclusions

Conclusions

- Covariance stationary VAR: properties
- Forecasting using VARs
- Granger causality test
- The impulse response function
- Structural VARs

Thank you

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L8 – Evaluating Regression Models

Fazeer Rahim (IMF East AFRITAC)

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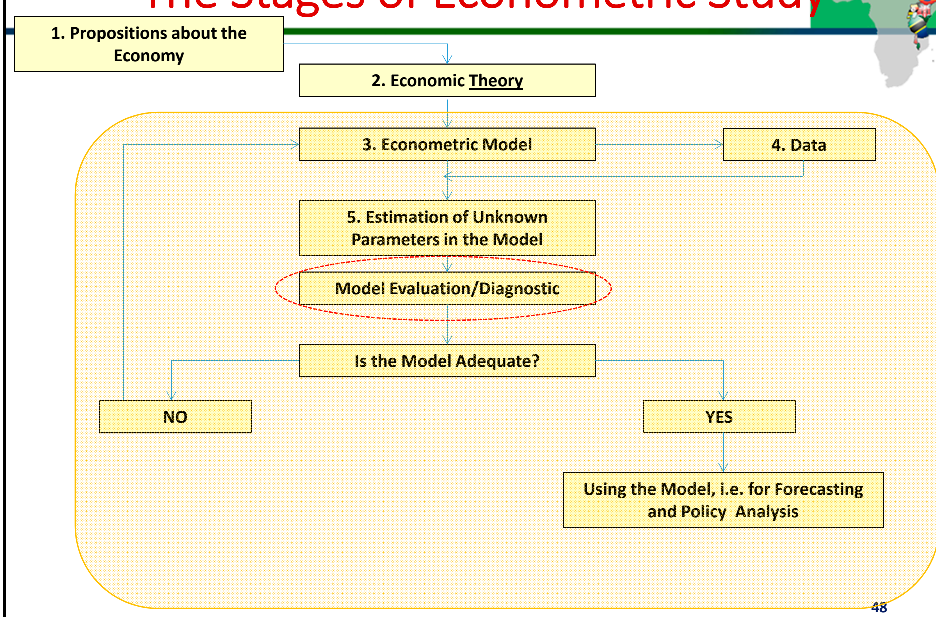
Objectives

I. Learn how to evaluate a regression model

- Residual tests
- Stability tests
- Coefficient tests

The Stages of Econometric Study



Multiple Linear Regression Model

- Multiple linear regression model

$$Y_i = f(x_{i1}, x_{i2}, \ldots, x_{ik}) + \varepsilon_i = \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_k x_{ik} + \varepsilon_i,$$
$$i = 1, \ldots, n$$

- Y is the ***dependent*** or ***explained*** variable
- $x_{i1}, x_{i2}, \ldots, x_{ik}$ are the ***independent*** or ***explanatory*** variables
- **The theory specifies** $f(x_{i1}, x_{i2}, \ldots, x_{ik})$
- ε is a **random disturbance**: it **"disturbs" and otherwise stable relationship**
- We assume that each observation in a sample is generated by the underlying process described by the function ***f***
- **The objective: estimate the parameters *β* of the model**

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Measuring Independent Effects

The usefulness of multiple regression models

- Multiple regression models can **isolate independent effects** of a set of variables on a dependent variable
- Example:

  Are higher levels of education associated with higher incomes?

  Income = β1+ β2 education + ε

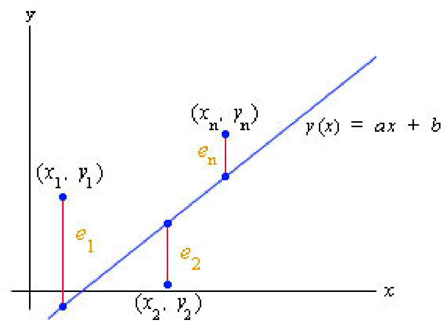
  But most people have higher income when they have more experience, independent of their education. B2 could be overstated: if experience and education are positive correlated, the model associate all the increases to income from increases in education.

  Better specification would account for experience:

  Income = β1+ β2 education + β3 experience + ε

Ordinary Least Squares

- Ordinary least squares (OLS) or linear least squares is a method for estimating the unknown parameters in a linear regression model
- The least squares best fit is found by finding the **minimum value of the square of errors**.
  - *In example below: find a and b such as sum of squared errors are minimum*



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Best Linear Unbiased Estimator

- The **Gauss-Markov Theorem** states that, provided the **Classical assumptions hold**, the ordinary least squares (OLS) *estimator b is the minimum variance estimator among all linear unbiased estimators*.
- Some- times it is said that the OLS estimator is BLUE (**Best Linear Unbiased Estimator**).
- For this to happen, our model needs to fulfil the following conditions:

1. Linearity
2. Expected value of error is zero
3. X and errors are uncorrelated
4. Absence of serial autocorrelation
5. Errors are homoskedastic

If Assumptions 1,2, 3 are satisfied, then the least squares estimator of the regression coefficients is ***unbiased***. If assumption 4-5 are satisfied, OLS has the **minimum variance** among all estimators

Assumptions of the Classical Linear Model

1. ***Linear functional form of the relationship Y and X***

$Y=\alpha+\beta x+\varepsilon$ OK

$Y=\alpha+x^{\beta}+\varepsilon$ NOT OK

$Y=Ax^{\beta}\varepsilon$ $\longrightarrow$ $\log Y=\mathrm{Log}A+\beta\log x+\varepsilon$ **OK**

2. ***Expected value of the disturbance is zero*** at every observation, i.e. disturbances are random

If we expected a particular error value, then (part of) the error term would be predictable, and we could just add that to the regression model.

3. ***X and ε are uncorrelated*** *($Cov(X_i,\varepsilon_i) = 0$), i.e X is exogenous*

When assumption is violated, we say Xi is endogenous.

Why is endogeneity a problem?

We are attributing variation in Yi to Xi that is really due to εi varying with Xi*. Consequently, we get a biased estimate of the coefficient on X, β, because it measures the effect of X and ε on Y.*

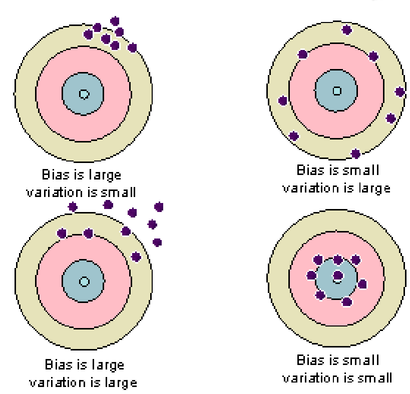
4. No serial autocorrelation, that is errors are statistically independent: *$E\{\varepsilon_i'\varepsilon_j\}=0$* for any *i, j*

Assumptions of the Classical Linear Model (continued)

5. ***Homoskedasticity****:*
Variance of disturbances is constant

The least squares estimator is unbiased even if this assumption is violated*.*

But*, it turns out* ***there are more efficient estimators (i.e. smaller variance) than least squares if the errors are heteroskedastic.***



Accuracy versus Quality of an Estimator Using Bias and Variation as Measurable Quantities Respectively

Source: http://home.ubalt.edu/ntsbarsh/business-stat/BiasVaraince.gif, *acessed on May 07 2015*

Two more assumptions

6. No perfect collinearity
With perfect collinearity, one (or more) independent variables is a perfect linear function of others. Perfect collinearity is a problem, because the least squares estimator cannot separately attribute variation in Y to the independent variables.

7. Normality of errors
In additional, the errors (disturbances) should be normally distributed, with zero mean and constant variance.
Important for hypothesis testing.

Goodness of Fit and Analysis of Variance

- The **goodness of fit** of a statistical model describes **how well it fits a set of observations**. A commonly used measure of the goodness of fit is the **coefficient of determination, the R^2**.
- Computation is based on the analysis of variance procedure that partitions the **total variation in the dependent variable**, denoted **SST** (total sum of squares), into two parts: the part explained by the estimated regression equation, denoted **SSE**, and the part that remains unexplained, denoted **SSR**
- *SSR + SSE = SST*
- **R^2 =SSR/SST, and varies between 0 and 1, where 1 means that SSR is equal to SST**

Note: R-squared *cannot* determine whether the coefficient estimates and predictions are biased, which is why you must assess the residual plots.

Hypothesis Testing

- In a regression study, hypothesis **tests are usually conducted to assess the statistical significance** of the overall **relationship represented by the regression** model and to test for the statistical **significance of the individual parameters**.
- The statistical **tests used are based on the assumptions concerning the disturbances**
- If the overall model is deemed statistically significant, conduct **hypothesis tests on the individual parameters to determine if each independent variable makes a significant contribution to the model**

Summarizing the discussion so far

- What are we looking for when construct and evaluate a regression?
- Before using our model for the purpose of forecasting, we would like
  1. to find "right" factors (regressors) X that "explain" process Y and thus have unbiased estimates of the model parameters and have minimum variance of our residual.
  2. our model to have parameters that are stable over time
     - "stable" - should not change wildly over time
     - or at least we should be able to detect breaks so that we can "deal" with them
  3. to check hypotheses about our model
- The tools that we have – TESTS:
  - Standard and rely on certain (calculated) test-statistics
  - Idea: compare a value of a test statistic to a critical value(s): if the test statistic exceeds the critical value(s), then a tested hypothesis is rejected

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Example

- We introduce an example of a linear regression model to see how we can evaluate models using
  1. Residual Tests
  2. Stability Tests
  3. Coefficient Tests

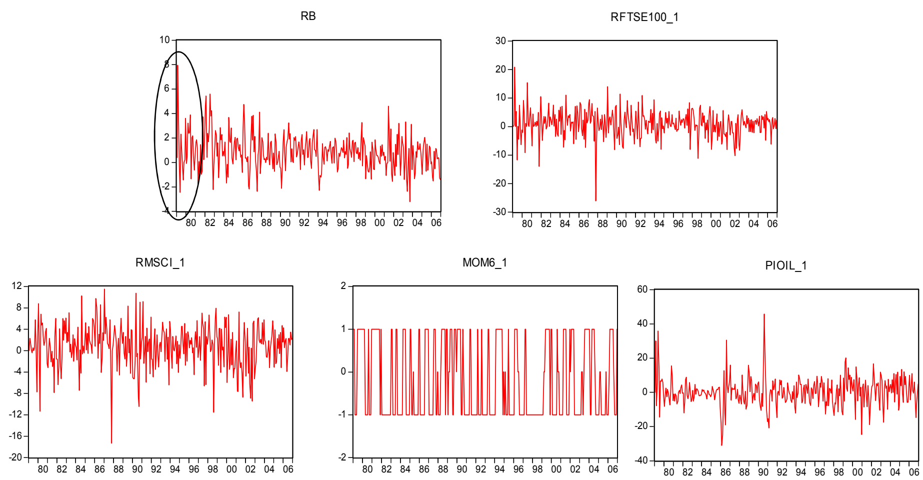
  … in practice

Example: UK Bond Returns Forecasting Model

- Regression model for rB_t – return on a "portfolio" of long-term bonds over one month
- Full sample: Jan. 1979-Dec. 2006
- At time t we use available data to predict rB_{t+1}
- Factors:
  - Bond returns at lags 1 and 2: rB_{t-1}, rB_{t-2}
  - Return on the FTSE100 stock index: $rftse_{t-1}$
  - Return on the MSCI world stock index: $rmsci_{t-1}$
  - Momentum: $mom6_{t-1}$
  - Change in the price for oil: $pioil_{t-1}$
  - A dummy to account for an outlier in October 1979: *d1979_10*

Example: UK Bond Returns Forecasting Model

- **Before estimating:** have a look at the series!



Example: UK Bond Returns Forecasting Model

- Initial Model:

$$rB_t = \beta_1 + \beta_2 rB_{t-1} + \beta_3 rB_{t-2} + \beta_4 rftse_{t-1} + \beta_5 rmsci_{t-1} + \dots + \beta_6 mom_{t-1} + \beta_7 \pi oil_{t-1} + \beta_8 d1979\_10 + \varepsilon_t$$

- Full sample OLS-estimation:

Dependent Variable: RB
Method: Least Squares
Date: 03/16/12 Time: 11:24
Sample (adjusted): 1979M03 2006M12
Included observations: 334 after adjustments

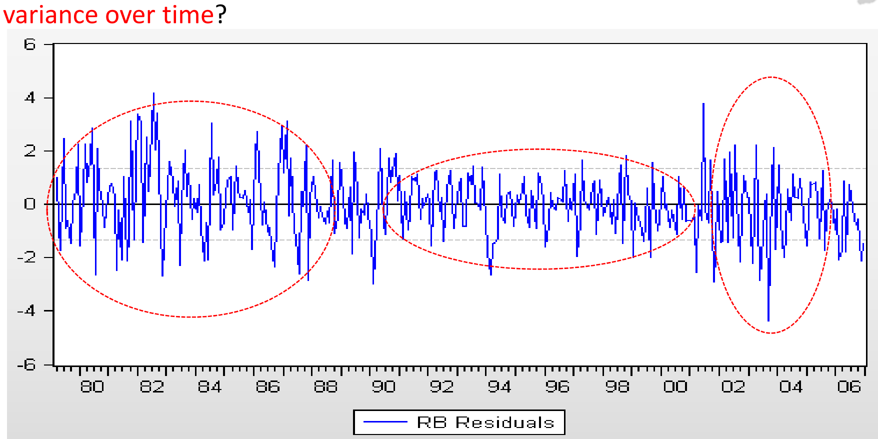
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.740048	0.092219	8.024889	0.0000
RB(-1)	0.248884	0.059136	4.208710	0.0000
RB(-2)	-0.127012	0.054603	-2.326084	0.0206
RFTSE100_1	-0.014633	0.019286	-0.758770	0.4485
RMSCI_1	-0.030721	0.022615	-1.358444	0.1753
MOM6_1	-0.109447	0.098349	-1.112841	0.2666
PIOIL_1	-0.026694	0.009036	-2.954260	0.0034
D1979_10	-5.187950	1.341878	-3.866187	0.0001

R-squared	0.163789	Mean dependent var	0.801884
Adjusted R-squared	0.145833	S.D. dependent var	1.442788
S.E. of regression	1.333441	Akaike info criterion	3.437064
Sum squared resid	579.6494	Schwarz criterion	3.528348
Log likelihood	-565.9896	Hannan-Quinn criter.	3.473460
F-statistic	9.121955	Durbin-Watson stat	1.979125
Prob(F-statistic)	0.000000		

Residual Tests

Eyeballing the residuals

- What can you say about the mean of the residuals, serial correlation, and the variance over time?



- Get suspicious: non-constant variance - heteroskedasticity? Not **efficient** estimators

Residual Tests

- To assess the validity of assumptions, we need Residual Tests
  - Residual autocorrelation. Are errors independent: $E\{\varepsilon_i'\varepsilon_j\}=0$ for any i, j?
  - Homoskedasticity. Is the error variance constant: $Var\{\varepsilon\}=E\{\varepsilon'\varepsilon\}=\sigma^2$?
  - Normality. Are residuals normally distributed:

$$\hat{\varepsilon}_t \approx N\left(0, \sigma^2\right)$$

Autocorrelation: *Q*-Test and LM Tests

1. The **Q-statistic** looks at the correlogram of the residuals

Null hypothesis: data is independently distributed (correlation 0)

Intuition of Q test:

- if residuals are autocorrelated, then *Q(m)* should be "large"
- If *Q(m)* is "large enough" - larger than a "critical value" - then H_0 is rejected (so, residuals are autocorrelated indeed)

If residuals are autocorrelated, then *Q(m)* should be "large"

- If *Q(m)* is "large enough" - larger than a "critical value" - then H_0 is rejected (so, residuals are autocorrelated indeed)

2. LM-tests:

- Estimate a model with OLS and get residuals: $\hat{\varepsilon}_t = y_t - \hat{\beta}_1 - \hat{\beta}_2 x_{2t} - \hat{\beta}_3 x_{3t} - \ldots - \hat{\beta}_k x_{kt}$
- Regress residuals on *X*'s and lagged residuals;

$$\hat{\varepsilon}_t = \gamma_1 + \gamma_2 x_{2t} + \ldots + \gamma_k x_{kt} + \varphi_1 \hat{\varepsilon}_{t-1} + \ldots + \varphi_m \hat{\varepsilon}_{t-m} + \nu_t$$

- Test H_0: $\varphi_1=\varphi_2=\ldots=\varphi_m=\gamma_1=\gamma_2=\ldots=\gamma_k=0$

- **Rejecting H0 means: autocorrelation**

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Residual Tests: Tests for Autocorrelation

- In Eviews:

Correlogram of Residuals

Date: 03/16/12 Time: 11:33
Sample: 1979M03 2006M12
Included observations: 334

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.008	0.008	0.0205	0.886
		2	0.022	0.022	0.1805	0.914
		3	0.059	0.058	1.3414	0.719
		4	-0.028	-0.029	1.6040	0.808
		5	-0.039	-0.041	2.1151	0.833
		6	0.007	0.005	2.1303	0.907
		7	-0.018	-0.013	2.2406	0.945
		8	-0.002	0.002	2.2414	0.973
		9	0.075	0.074	4.2018	0.898
		10	-0.021	-0.022	4.3565	0.930
		11	0.100	0.098	7.8594	0.726
		12	-0.059	-0.071	9.0522	0.698

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.970504	Prob. F(12,314)	0.4770
Obs*R-squared	11.94481	Prob. Chi-Square(12)	0.4501

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Sample: 1979M03 2006M12
Included observations: 334

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.176365	0.151583	1.163486	0.2455
RB(-1)	-0.013273	0.136194	-0.097459	0.9224
RB(-2)	-0.196033	0.132516	-1.479316	0.1401
RFTSE100_1	-0.001508	0.019977	-0.075488	0.9399
RMSCI_1	-0.003937	0.023275	-0.169174	0.8658
MOM6_1	0.008353	0.104921	0.079614	0.9366
PIOIL_1	0.000273	0.009246	0.029486	0.9765
D1979_10	0.070177	1.357996	0.051677	0.9588
RESID(-1)	0.034507	0.144754	0.238384	0.8117
RESID(-2)	0.220415	0.138156	1.595403	0.1116
RESID(-3)	0.122797	0.069138	1.776106	0.0767
RESID(-4)	-0.028536	0.058527	-0.487561	0.6262
RESID(-5)	-0.043298	0.056985	-0.759813	0.4479
RESID(-6)	0.006884	0.057000	0.120779	0.9039
RESID(-7)	-0.008571	0.056604	-0.151421	0.8797
RESID(-8)	-0.002875	0.056671	-0.050731	0.9596
RESID(-9)	0.075229	0.056407	1.333690	0.1833
RESID(-10)	-0.022066	0.057380	-0.384549	0.7008
RESID(-11)	0.104312	0.057199	1.823670	0.0692
RESID(-12)	-0.072324	0.057096	-1.266709	0.2062

R-squared	0.035763	Mean dependent var	1.85E-16
Adjusted R-squared	-0.022583	S.D. dependent var	1.319352

- Question: Are errors autocorrelated?
- Answer: Both tests fail to reject H_0 of no autocorrelation: both Q-stat. and TR^2 do not exceed critical values (we can claim it because all p-values >5%)

What to do when errors are serially correlated?

- Solution:
  - Add missing variables
  - Add lagged dependent (y) and independent variables (x)

Residual Tests: Tests for Homoskedasticity

- **Breusch-Pagan-Godfrey Test**
  - Auxiliary model: $\hat{\varepsilon}_t^2 = \gamma_1 + \gamma_2 x_{2t} + \ldots + \gamma_k x_{kt} + v_t$
  - H_0: $\gamma_2 = \gamma_3 = \ldots = \gamma_k = 0$
  - Test statistic $T{\cdot}R^2 \sim \chi^2(k\text{-}1)$
- **White's Test**
  - Auxiliary model: $\hat{\varepsilon}_t^2 = \gamma_1 + \gamma_2 x_{2t} + \ldots + \gamma_k x_{kt} + squares + crossprod. + v_t$
  - H_0: $\gamma_2 = \ldots = \gamma_k = \lambda_1 = \lambda_2 = \ldots = \lambda_k = 0$
  - Test statistic $T{\cdot}R^2 \sim \chi^2(2k+k(k\text{-}1)/2\text{-}1)$
- **ARCH LM-Test**
  - Auxiliary model: $\hat{\varepsilon}_t^2 = \omega_1 + \omega_2 \hat{\varepsilon}_{t-1}^2 + \ldots + \omega_m \hat{\varepsilon}_{t-m}^2 + v_t$
  - H_0: $\omega_2 = \ldots = \omega_m = 0$
  - Test statistic $T{\cdot}R^2 \sim \chi^2(m\text{-}1)$
- **Intuition: if there is a systematic relation of $\hat{\varepsilon}_t^2$ and some regressors, then R^2 is going to be "large". If, as a result, TR^2 is larger than a critical value (significant), then we reject H_0 of homoskedasticity (errors are heteroskedastic)**

Residual Tests: Tests for Homoskedasticity

- In Eviews:

Correlogram of Residuals Squared

Date: 03/16/12 Time: 11:48
Sample: 1979M03 2006M12
Included observations: 334

	AC	PAC	Q-Stat	Prob
1	0.090	0.090	2.7029	0.100
2	0.157	0.150	10.995	0.004
3	0.196	0.176	24.080	0.000
4	0.145	0.105	31.251	0.000
5	0.080	0.016	33.409	0.000
6	0.102	0.034	36.961	0.000
7	0.137	0.082	43.369	0.000
8	0.025	-0.031	43.590	0.000
9	0.112	0.059	47.919	0.000
10	0.068	0.014	49.514	0.000
11	0.105	0.063	53.331	0.000
12	-0.018	-0.074	53.449	0.000

Heteroskedasticity Test: Breusch-Pagan-Godfrey

F-statistic	2.767163	Prob. F(7,326)	0.0083
Obs*R-squared	18.73244	Prob. Chi-Square(7)	0.0091
Scaled explained SS	20.29686	Prob. Chi-Square(7)	0.0050

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 03/16/12 Time: 11:50
Sample: 1979M03 2006M12
Included observations: 334

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.581281	0.178015	8.882833	0.0000
RB(-1)	0.302812	0.114152	2.652696	0.0084
RB(-2)	0.136317	0.105404	1.293284	0.1968
RFTSE100_1	-0.049956	0.037228	-1.341888	0.1806
RMSCI_1	-0.042978	0.043654	-0.984521	0.3256
MOM6_1	0.553875	0.189849	2.917455	0.0038
PIOIL_1	-0.031091	0.017442	-1.782519	0.0756
D1979_10	-1.436644	2.590299	-0.554625	0.5795

R-squared	0.056085	Mean dependent var	1.735477
Adjusted R-squared	0.035817	S.D. dependent var	2.621387
S.E. of regression	2.574014	Akaike info criterion	4.752471
Sum squared resid	2159.928	Schwarz criterion	4.843755
Log likelihood	-785.6626	Hannan-Quinn criter.	4.788867
F-statistic	2.767163	Durbin-Watson stat	1.891509
Prob(F-statistic)	0.008314		

- Question? What would you say about homoskedasticity?
- Both tests reject H_0 of homoskedasticity: both Q-stat. and TR^2 exceed critical values (p-values < 5%)

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What to do when errors are heteroskedastic?

- Heteroskedasticity of errors:
  - OLS-estimators for parameters are still unbiased/consistent…But they are not efficient:
- Solution:
  - Use alternative estimation method: Generalized least square (GLS) – not explicit in Eviews
  - Use heteroskedasticity and autocorrelation consistent (HAC) estimator for the error variance – possible in EViews
  - Model the variance explicitly as an ARCH/GARCH process, estimate the model using Maximum Likelihood Estimation (MLE) – see online course for more details

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One solution to heteroskedasticity ARCH Models

ARCH models: The conditional variance of ε_t is dependent on the realized values of ε_{t-1}. If the realized values are large, the conditional variance in t will be large as well. Illustration using an ARCH(1). Note: α will determine the persistency of the shocks

$$E_{t-1}y_t = a_0 + a_1 y_{t-1}$$

and

$$Var(y_t \mid y_{t-1}, y_{t-2}, ...) = E_{t-1}(y_t - a_0 - a_1 y_{t-1})^2 = E_{t-1}(e_t)^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2$$

$$E\left[\varepsilon_t^2 / \varepsilon_{t-1}, \varepsilon_{t-2}, \varepsilon_{t-3}, ...\right] = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2$$

Residuals can come from an autoregressive process

The conditional heteroskedasticity of ε_t will result in y_t being heteroskedasticity itself. Thus the ARCH model is able to capture changes in volatility in the series yt

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ARCH Models - Generalization

A natural extension of **ARCH(1)** is a more general model with longer lags, **ARCH(q)**:

$$y_t = \mu + \sum_{j \in J} \rho_j y_{t-j} + \varepsilon_t$$

AR(J)-ARCH(q)

$$\varepsilon_t \sim N\left(0, \sigma_t^2\right)$$

$$\boxed{\sigma_t^2 = \omega + \sum_{i=1}^{i=q} \gamma_i \varepsilon_{t-i}^2}$$

ARCH(q)

Persistency of shocks will be determined by γs

Weighted average of past shocks (news) to current volatility

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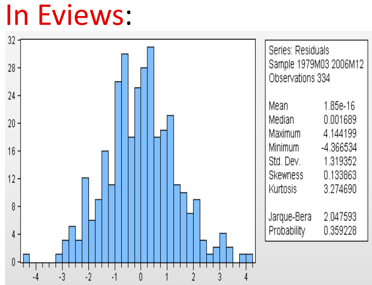
Residual Tests: Tests for Normality

- Jarque-Bera-test statistic:

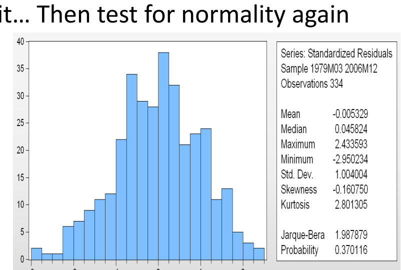
$$JB = \frac{T}{6}\left(S^2 + \frac{(K-3)^2}{4}\right) \to \chi^2(2)$$

where *S* is skewness and *K* is kurtosis.

- H_0: residuals are normal (*JB*~3)
- In Eviews:



CAREFUL!!! You found heteroskedasticity – model it… Then test for normality again



Question: Are residuals normal?

Answer: JB-test does not reject H_0 of normality: *JB*-stat. does not exceed critical value(*p*-value > 5%)

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Stability Tests

Stability Tests

- How stable is the model/model parameters over time? *In other words, is the structure of the data-generating process stable over time?*
- Testing Structural Change:
  - **Chow Breakpoint Test**: fit the same model to the pre-break data and the post-data. If the 2 models are not sufficiently different, then no structural change in the data generating process. Problem: there are structural changes that do not have a specific date
  - **Stability Test using Recursive Coefficient:** When no specific date can be identified, use **Recursive Tests**. Example: if you have 150 obs, estimate the model using a few (say 10 obs), plot the estimated coefficients, reestimate the model using 11 obs, plot the estimated coefficients. Keep repeating**, if the magnitude of a coefficient suddenly begins to change you have a break**.

Stability Tests: Chow Breakpoint Test

- Chow Breakpoint Test
  - Restricted Model: $y_t = x_t\beta + \varepsilon, \forall t = 1, \ldots T$
  - Unrestricted Model (test for *m*=1 breaks) $y_t = x_t\beta_1 + \varepsilon_1, \forall t = 1, \ldots, T_1$

$$y_t = x_t\beta_2 + \varepsilon_2, \forall t = T_1 + 1, \ldots, T$$

  - The test compares errors of a model fitted for the whole sample [1,*T*] to the errors when models are fitted to the separate subsamples [1, T_1], [T_1+1 T_2],..., [T_m+1,T]
  - H_0: there is no break
  - *F*-statistic:

$$F = \frac{\left(\hat{\varepsilon}_R{}'\hat{\varepsilon}_R - \hat{\varepsilon}_U{}'\hat{\varepsilon}_U\right)/mk}{\hat{\varepsilon}_U{}'\hat{\varepsilon}_U / \left(T - (m+1)k\right)} \approx F\left(mk, T - (m+1)k\right)$$

$$\hat{\varepsilon}_U{}'\hat{\varepsilon}_U = \sum_{i=1 \text{ to } m+1} \hat{\varepsilon}_{iU}{}'\hat{\varepsilon}_{iU}, \text{ where } i \text{ corresponds to a subsample}$$

  - **Intuition: if the U-model (that accounts for a break) produces a sum of squared errors that is substantially smaller (so that there is a break indeed) than that for the R-model, then *F*-stat. is going to be "large"…**

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Stability Tests: Chow Forecasting Test

Chow Forecast Test

Full-Sample Model: $y_t = x_t\beta + \varepsilon, \forall t = 1, \ldots T$

Sub-Sample Model: $y_t = x_t\beta_1 + \varepsilon_1, \forall t = 1, \ldots, T_1, \quad T_1 < T$

The test compares in-sample residuals of the 2 models

H_0: there is no break

F-statistic:

$$F = \frac{\left(\hat{\varepsilon}_{FS}{}'\hat{\varepsilon}_{FS} - \hat{\varepsilon}_{SS}{}'\hat{\varepsilon}_{SS}\right)/(T - T_1 + 1)}{\hat{\varepsilon}_{SS}{}'\hat{\varepsilon}_{SS} / (T_1 - k)} \approx F\left(T - T_1 + 1, T_1 - k\right)$$

where $\hat{\varepsilon}_{FS} = y_t - x_t\hat{\beta}$ for $t = 1, \ldots, T$ and $\hat{\varepsilon}_{SS} = y_t - x_t\hat{\beta}_1$ for $t = 1, \ldots, T_1$

Intuition: if there is no break then the fit in both the full-sample and the sub-sample should be of the same degree of accuracy . If there is a break, then the fit of the full-sample model should deteriorate such that $\hat{\varepsilon}_{FS}{}'\hat{\varepsilon}_{FS}$ becomes "large", which induces the *F*-statistic to become "large"…

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Stability Tests: Chow Tests

- Eviews illustration of the CBT and CFT: is there a break in December 1995?

Chow Breakpoint Test

Null Hypothesis: No breaks at specified breakpoints
Varying regressors: All equation variables
Equation Sample: 1979M03 2006M12

F-statistic	1.709453	Prob. F(4,323)	0.1476
Log likelihood ratio	6.934032	Prob. Chi-Square(4)	0.1394
Wald Statistic	6.837814	Prob. Chi-Square(4)	0.1447

Chow Forecast Test

Equation: UNTITLED
Specification: RB01 C RB01(-1) RB01(-2) PIOIL_1
Test predictions for observations from 1995M12 to 2006M12

	Value	df	Probability
F-statistic	0.778471	(133, 194)	0.9390
Likelihood ratio	141.5618	133	0.2895

F-test summary:

	Sum of Sq.	df	Mean Squares
Test SSR	166.6440	133	1.252962
Restricted SSR	478.8904	327	1.464497
Unrestricted SSR	312.2464	194	1.609518
Unrestricted SSR	312.2464	194	1.609518

LR test summary:

	Value	df
Restricted LogL	-530.7966	327
Unrestricted LogL	-460.0157	194

Unrestricted log likelihood adjusts test equation results to account for observations in forecast sample

- Question: Can you reject the hypothesis of a structural break in 1995M12?
- Answer: Both tests do not reject H_0 of no structural break in 1995m12
- **Note: to run the stability tests the model should be re-estimated using Weighted OLS with weights brought by the estimate of errors std. deviation to adjust for the heteroskedasticity (more on this in the workshop).**

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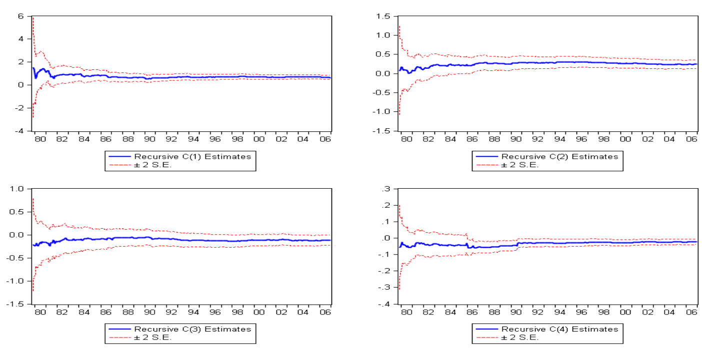
Stability Tests: Recursive Coefficients

- Recursive Coefficients
- The regression model is recursively estimated on m subsamples $[1, T_1]$, $[1, T_1+1]$, …, $[1, T]$ to produce a sequence of $\hat{\beta}_m$

$$y_m = x'_m \beta_m + \varepsilon_m$$

- If it is found that the value of (some) estimated coefficients change wildly over time, then it indicates breaks
- Eviews example:

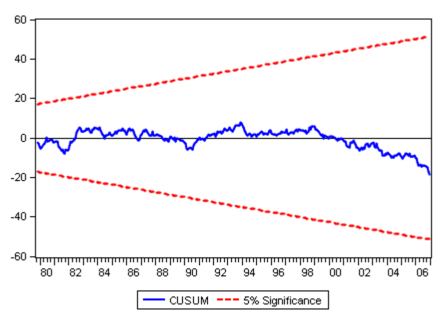


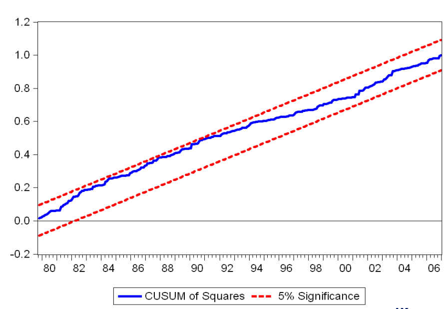
Stability Tests using Recursive coefficients: CUSUM and CUSUM of Squares Test

- The CUSUM tests are based on the cumulative sum of the recursive residuals
- The test finds parameter instability if the cumulative sum goes outside the area between the two critical lines

Stability Tests: CUSUM and CUSUM of Squares Test

- The CUSUM test rejects stability at 5% significance level if the test statistics W_t crosses limits given by $\pm 0.948[\sqrt{T-k} + 2(t-k)\sqrt{T-k}]$
- The CUSUM of Squares test rejects stability if the test statistic S_t crosses limits given by $\pm c + (t-k)/(T-k)]$, where *c* depends on the chosen significance level
- Eviews example:





Hypothesis Testing

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Hypothesis Testing: Coefficient Tests

- We use these tests to evaluate hypotheses about parameter values:
  - should a variable be excluded from a regression – a corresponding parameter value is (statistically) zero?
  - could a certain parameter(s) be restricted to a particular economic-theory-consistent value(s)?
- Coefficient Tests we are looking at here:
  - Test for significance of a particular coefficient: *t*-test
  - Test of linear restrictions for coefficients: Wald test (restricted comes from the idea of a restricted model, fewer variables)
  - Test for omitted/redundant variables: Log-likelihood ratio test

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Significance of a particular coefficient

- Simple univariate model:

$$y_t = \beta x_t + \varepsilon_t$$

- We want to know if factor x_t is important (statistically significant) for explaining y_t: If $\hat{\beta} = 0$ then x_t is not important
- t-statistic for a t-test:

$$t_{\hat{\beta}} = \frac{\hat{\beta} - \beta}{s.e.(\hat{\beta})} = \frac{\hat{\beta} - 0}{s.e.(\hat{\beta})} = \frac{\hat{\beta}}{s.e.(\hat{\beta})}$$

- Logic: if $\hat{\beta}$ is far from 0, then $t_{\hat{\beta}}$ is going to be either quite large (if $\beta > 0$) or quite small (if $\beta < 0$)
- Also, for $t_{\hat{\beta}}$ to be valid it is rather important that $\hat{\beta}$ and $s.e.(\hat{\beta})$ are "correct" (e.g. have "good" properties – consistent estimates of true values)
- Properties of model errors are key for desirable properties of parameters' estimators and for correct implementation of model diagnostics!!!

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Coefficient Tests: Wald Test

- **To test whether two or more coefficients within a model are equal, if we can exclude variables from our model**
- Our model in a post-residual-tests representation:

Dependent Variable: RB
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 04/04/12 Time: 17:59
Sample (adjusted): 1979M03 2006M12
Included observations: 334 after adjustments
Failure to improve Likelihood after 35 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(9) + C(10)*RESID(-1)^2 + C(11)*RESID(-2)^2 + C(12)*RESID(-3)^2 + C(13)*RESID(-4)^2

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.777527	0.085817	9.060317	0.0000
RB(-1)	0.178766	0.059147	3.022404	0.0025
RB(-2)	-0.182484	0.055411	-3.293305	0.0010
RFTSE100_1	-0.005591	0.016372	-0.341516	0.7327
RMSCI_1	-0.018831	0.018685	-1.007800	0.3136
MOM6_1	-0.151719	0.081079	-1.871243	0.0613
PIOIL_1	-0.025090	0.006935	-3.617846	0.0003
D1979_10	-5.312879	165.8494	-0.032034	0.9744
Variance Equation				
C	0.751270	0.158691	4.734178	0.0000
RESID(-1)^2	0.115490	0.088904	1.299040	0.1939
RESID(-2)^2	0.141085	0.090357	1.561413	0.1184
RESID(-3)^2	0.288561	0.089664	3.218248	0.0013
RESID(-4)^2	0.041468	0.057173	0.725308	0.4683

R-squared	0.153936	Mean dependent var	0.801884
Adjusted R-squared	0.135769	S.D. dependent var	1.442788
S.E. of regression	1.341274	Akaike info criterion	3.368548
Sum squared resid	586.4791	Schwarz criterion	3.516886
Log likelihood	-549.5476	Hannan-Quinn criter.	3.427693
Durbin-Watson stat	1.865530		

- We test if coefficients for $mom6_t$, $rftse_t$, $rmsci_t$ and are jointly statistically insignificant. IN EVIEWS, we need to put the following restriction: c(4)=c(5)=c(6)=0 in Eviews

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Coefficient Tests: Wald Test

- Eviews illustration of Wald test for the *rB* regression

Dependent Variable: RB
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 04/04/12 Time: 10:11
Sample (adjusted): 1979M03 2006M12
Included observations: 334 after adjustments
Failure to improve Likelihood after 35 iterations
Presample varian
GARCH = C(9) + C
-3)^2 + C(13)*

Wald Test
Coefficient restrictions separated by commas
c(4)=c(5)=c(6)=0
Examples
C(1)=0, C(3)=2*C(4)
OK Cancel

Variable				
C				00
RB(-1)				25
RB(-2)				10
RFTSE100_1				27
RMSCI_1				36
MOM6_1				13
PIOIL_1				03
D1979_10	-5.312879	165.8494	-0.032034	0.9744

Variance Equation

C	0.751270	0.158691	4.734178	0.0000
RESID(-1)^2	0.115490	0.088904	1.299040	0.1939
RESID(-2)^2	0.141085	0.090357	1.561413	0.1184
RESID(-3)^2	0.288561	0.089664	3.218248	0.0013
RESID(-4)^2	0.041468	0.057173	0.725308	0.4683

...

Wald Test:
Equation: EQ01_INITIAL_OLS

Test Statistic	Value	df	Probability
F-statistic	1.995928	(3, 321)	0.1145
Chi-square	5.987784	3	0.1122

Null Hypothesis: C(4)=C(5)=C(6)=0
Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(4)	-0.005591	0.016372
C(5)	-0.018831	0.018685
C(6)	-0.151719	0.081079

Restrictions are linear in coefficients.

- Question: Would you be able to reject the null hypothesis?
- Answer: We do not reject the null of coefficients being 0 jointly

Coefficient Tests: Log-Likelihood Ratio Test

- Omitted Variables Test: we start from a Restricted model
- Redundant Variables Test: we start from an Unrestricted model
- The H_0 is that redundant/omitted variables do not belong to a U-model
- Test statistic in either case – the Likelihood Ratio:

$$LR = -2\left(\ln L_R - \ln L_U\right) \xrightarrow{A} \chi^2(r),$$

where *r* is a number of omitted variables

- Intuition:
  - Omitted variable test: if adding an extra variable (going from R to U) improves the log-likelihood substantially (makes *LR* large), then this variable should be in the model – we reject the null
  - Redundant variable test: if dropping a variable (going from U to R) decreases the log-likelihood substantially (makes LR smaller), then this variable should not be excluded from the model – we reject the null

Coefficient Tests: LLR-test

- Eviews illustration of the Redundant variable LR-test for the *rB* regression

Dependent Variable: RB
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 04/04/12 Time: 10:20
Sample (adjusted): 1979M03 2006M12
Included observations: 334 after adjustments
Failure to improve Likelihood after 35 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(9) + C(10)*RESID(-1)^2 + C(11)*RESID(-2)^2 + C(12)*RESID(-3)^2

Redundant Variables Test

One or more test series to remove

RFTSE100_1 RMSCI_1 MOM6_1

OK Cancel

Va	tic	Prob.
	17	0.0000
R	04	0.0025
R	05	0.0010
RFTS	16	0.7327
RM	00	0.3136
MC	43	0.0613
PI	46	0.0003
D19	34	0.9744

Variance Equation

C	0.751270	0.158691	4.734178	0.0000
RESID(-1)^2	0.115490	0.088904	1.299040	0.1939
RESID(-2)^2	0.141085	0.090357	1.561413	0.1184
RESID(-3)^2	0.288561	0.089664	3.218248	0.0013
RESID(-4)^2	0.041468	0.057173	0.725308	0.4683

Redundant Variables Test
Equation: EQ01_INITIAL_OLS
Specification: RB C RB(-1) RB(-2) RFTSE100_1 RMSCI_1 MOM6_1 PIOIL_1 D1979_10
Redundant Variables: RFTSE100_1 RMSCI_1 MOM6_1

	Value	df	Probability
Likelihood ratio	5.198646	3	0.1578

LR test summary:

	Value	df
Restricted LogL	-552.1469	324
Unrestricted LogL	-549.5476	321

Conclusion

- Model evaluation is an important part of modeling business
- Should start with testing of residuals to see if all assumptions are valid
- If they are – congratulations!!! – you can proceed with the OLS
- If they are not – more work is needed:
  - reformulation
  - alternative estimation methods, etc.
- Further model evaluation: coefficient hypothesis testing and stability testing

Thank you

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Workshop on Macro-Fiscal Forecasting
Arusha, August 8 – 18, 2016

L9. The Effective Tax Rate Approach

Phyllis Resnick (Consultant)

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Outline

I. Introduction: revenues, baseline projections, and types of forecasts

II. The Effective Tax Rate approach: definition and use

III. Making assumptions about the Effective Tax Rate

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Part 1. Introduction: revenues, baseline projections, and types of forecasts

Revenues and expenditures

Revenues

Non repayable receipts (i.e. receipts which do not give rise to an obligation of repayment):

Expenditures

Payments that do not (by themselves) decrease the stock of liabilities (amortization of debt is not an expenditure)

Financing

Non repayable receipts: taxes, profits, and grants

–

Expenditures

=

OVERALL BALANCE = FINANCING:
Domestic borrowing
- Central bank (monetization)
- Bank financing
- Non-bank financing

Foreign borrowing

Privatization receipts

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Objective of Forecasting

Inform policymakers of fiscal effects of policy changes and/or of economic developments

"BASELINE SCENARIO"

- Assume current policies or confirmed policy changes
- Use macroeconomic projections at current policies

"POLICY SCENARIO"

- Simulate effect of changes to policies
- Use macroeconomic projections consistent with assumed policy change

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Forecasts Serve Multiple Purposes

Inform policymakers of fiscal effects of policy changes and/or of economic developments

Multiple Purposes and Time Horizons

- Budgeting – Generally 1 or 2 year horizon
- Revenue Monitoring - < 1 year horizon
- Planning – Horizon can be 10 years or longer

Focus here is on shorter time horizons

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Definition of Revenues

Non repayable receipts except grants and transfers (they do not give rise to an obligation of repayment)

Total Revenues and Grants
Tax revenues
Direct taxes
Taxes on income
Taxes on wealth
Indirect taxes
Taxes on goods and services (VAT, sales tax, turnover)
Taxes on imports
Other tax revenues
Nontax revenue
License, fees, etc.
Central Bank profits
Grants and transfers

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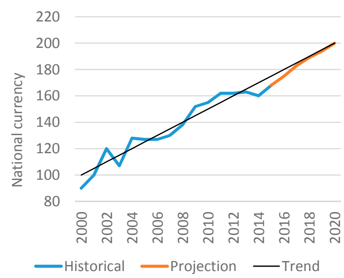
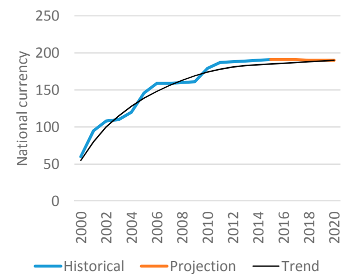
Unconditional forecasting methods

Use only past dynamic of the variable to project the future dynamic (example, ARIMA models)

$$y_t = a + b_1 y_{t-1} + b_2 y_{t-2} + \ldots + b_p y_{t-p} + \varepsilon_t$$





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Conditional forecasting methods

Use the past dynamic of the variable and its relation with other variables to project the future dynamic

- Econometric models (distributed lag models)
- Microeconomic models (using data from actual tax returns to simulate the effects of changes to rates or the effect of economic developments on revenues)
- Effective tax rate approach
- Elasticity approach

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Which method to use?

There is no "best" method

Which method to use depends on:

- The question
- Data availability
- The resources available (time, software, capacity)

Use more than one method, prefer simplicity first, and use best judgment

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Part 2. The Effective Tax Rate approach: definition and use

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How tax revenues are originated

Tax revenue = Statutory rate * Tax base

TAX BASE: the event or condition that gives rise to taxation and is defined in the law.

- Taxable event: Receipt of wages, sale of goods
- Taxable condition: Ownership of a house

STATUTORY RATE: the percent of tax base (for taxes *ad-valorem*) or the fixed amount per unit (per unit taxes) that must be paid

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Forecasting based on statutory tax rates

…requires a lot of information: tax rates and brackets, income distribution, deductions

From tax code			
Income brakets	< 60	60 - 100	>100
Statutory tax rate	15%	30%	50%

Income 2015	Person 1	Person 2	Person 3	Total
Taxable income	20	40	120	180
Applicable rate	15%	15%	50%	---
Personal tax	3	6	60	69

Income 2016	Person 1	Person 2	Person 3	Total
Taxable income	20	70	90	180
Applicable rate	15%	30%	30%	---
Personal tax	3	21	30	54

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The Effective Tax Rate

Effective Tax Rate = Tax revenue / Proxy tax base

PROXY TAX BASE: Economic variable that is closely related to (but it is NOT) the actual tax base

EFFECTIVE TAX RATE: the ratio between the tax revenue and the proxy tax base

Note: ETR can be defined for total or a specific tax revenue

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Rationale for the ETR approach

Most commonly used revenue forecasting method:

- Data are available
- It is simple and tractable
  - No need to project numerous actual tax bases
  - No need to collect information on statutory tax rates and exemptions for various tax items
- Can be applied for meaningful tax aggregates
  - Actual tax base is defined at very detailed level while proxy can be defined for broad aggregates

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Forecasting using the ETR

Tax revenue = Assumed ETR * Forecast of proxy tax base

Main steps:

- Select the proxy tax base
- Forecast the proxy tax base
- Forecast the ETR
- Obtain the revenue forecast multiplying the forecast of the proxy tax base by the assumed ETR

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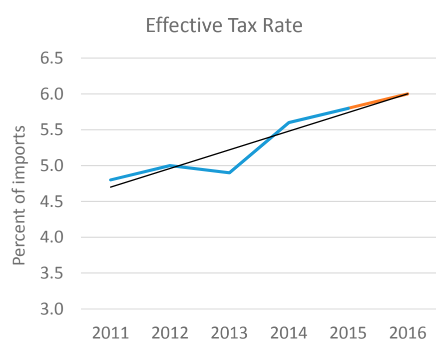
Example of ETR-based forecast

	2011	2012	2013	2014	2015	2016
Imports	16000	19000	25000	30000	35000	40000
ETR	4.8	5	4.9	5.6	5.8	6
Custom duties	768	950	1225	1680	2030	2400

6% : Trend ETR

It is important to ask…
- What do we assume?
- Is the assumption reasonable?



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Part 3. Making assumptions about the Effective Tax Rate

Choosing the proxy tax base

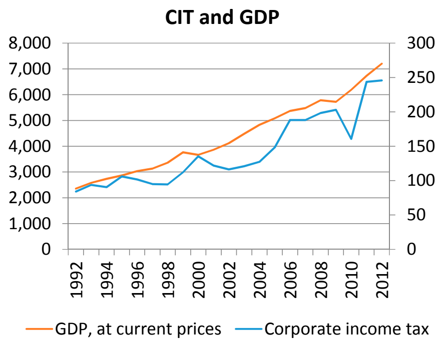
PROXY TAX BASE: Economic variable that is closely related to (but it is NOT) the actual tax base

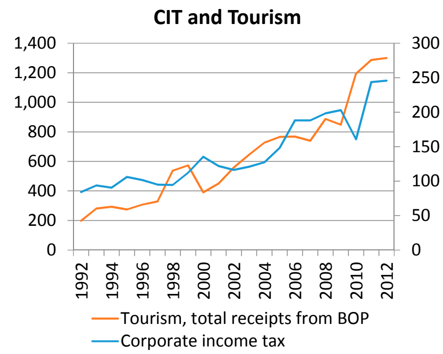
A good proxy tax base must be:

- Highly correlated with the actual tax base (either because of legal or economic reasons)
- Justifiable (the correlation can be explained either because of legal or economic reasons)
- Easy to forecast and link to main macroeconomic developments and macroeconomic policies

Example of a good/bad proxy (I)

The case of Fiji: tourism is one of the most important sectors of the economy, and the sector where most corporations (small/big) operate





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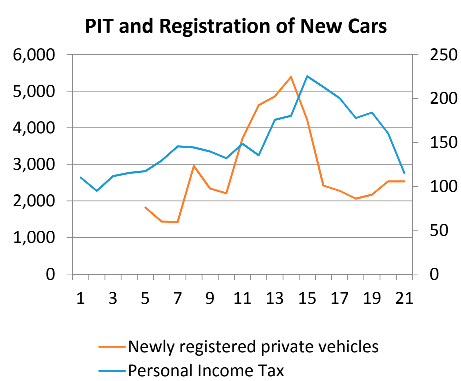
Example of a good/bad proxy (I)

Fiji: data on private consumption or household income are not good. However, registration of new cars is available…Why would new cars be a proxy base for PIT?



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Proxy tax bases

Useful proxies	
Individual income tax	Personal income, nominal GDP
Corporate income tax	Corporate profits, nominal GDP
VAT, Excise taxes, sale tax	Nominal private consumption, nominal GDP (imports or exports of goods and services)
Import duties	Imports
Export duties	Exports
Excises	Consumption of selected item, real private consumption expenditure, real GDP
Other taxes	Nominal GDP; production of natural resources
Non-tax revenues	Nominal GDP

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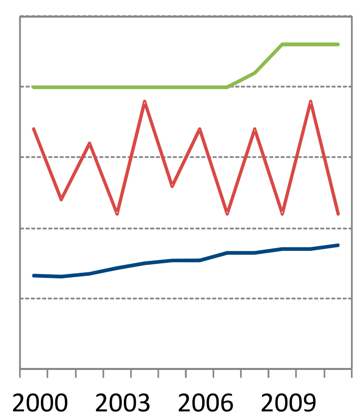
How to forecast the ETR (I)

1. Compute historical ETR
2. Make sense of past ETR:
   - Is it stable or unstable?
   - Does it follow a trend?
   - Have there been breaks?
   - What can explain past ETR?
3. Project…



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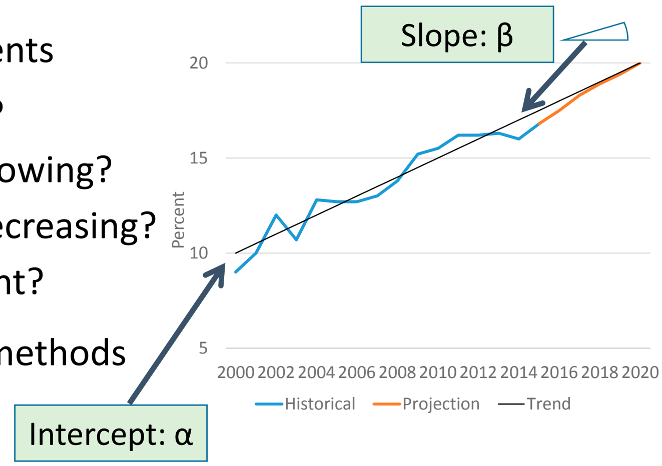
Projecting ETR

- Trend projection: $ETR_{year} = \alpha + \beta * (year)$
- Judgments
  - Period?
  - Ever-growing?
  - Ever-decreasing?
  - Constant?
- Other methods



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What could affect the ETR? (I)

THE STATUTORY TAX RATE, including exemptions, allowances, and other legal aspects defining how much should be paid

Statutory tax rate differs from the ETR reflecting...

- Proxy vs. actual tax base and aggregation
- Exemptions or deductions
- Illegal tax-free transaction
- Compliance rate

If the statutory rate increases or allowances decrease then the ETR would increase

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What could affect the ETR? (II)

COMPLIANCE RATE, gap between the actual revenue and the potential revenue

The compliance rate reflects:

- Effectiveness of tax administration
- Disincentives to evasion
- Incentives to comply (including simplicity of the tax)

If compliance improves, then the ETR increases

What could affect the ETR? (III)

BASE SHIFT, changes in the composition of the tax base

Base shifts generally reflect:

- Structural changes in the economy (example, greater importance of a sector that is taxed more heavily)
- Shocks

If the tax base shifts toward categories with higher statutory tax rate the ETR would increase

Tax base structure and ETR

Example: Income distribution

From tax code			
Income brakets	< 60	60 - 100	>100
Statutory tax rate	15%	30%	50%

Income 2015	Person 1	Person 2	Person 3	Total	ETR
Taxable income	20	40	120	180	---
Applicable rate	15%	15%	50%	---	38.3
Personal tax	3	6	60	69	---

Income 2016	Person 1	Person 2	Person 3	Total	ETR
Taxable income	20	70	90	180	---
Applicable rate	15%	30%	30%	---	30.0
Personal tax	3	21	30	54	---

Implication of assumptions about ETR

Constant ETR assumes

- Unchanged statutory tax rate (policy)
- Unchanged compliance rate (administration)
- Unchanged tax base structure (economic)

Keep in mind that ETR could increase reflecting…

- Statutory rate increase
- Administrative improvements
- Base shift from low- to high- tax category

Pros and cons of ETR approach

PROS	CONS
• Easy to use • Typically used for aggregates • Limited data needs • No need to keep track of statutory tax rates and exemptions for detailed tax items	• Need to use a lot of judgment (on effect of reforms, economic development, etc.) • Assumptions can be debatable (easy to hide unrealistic assumptions)

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Summary

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Conclusion

- No best forecasting methods: use more than one, but prefer simple methods first
- Effective Tax Rate approach: assume ETR and multiply by forecast of the (proxy) tax base
- To make assumptions about ETR study past ETR and explain changes (breaks) looking at changes in statutory rates, administrative changes, inflation, changes in the composition of the tax base

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L10. The Elasticity Approach

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Outline

I. The elasticity approach: definition and use

II. Interpreting elasticity and buoyancy

III. Computing buoyancy and estimating the elasticity

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Part 1. The elasticity approach: definitions and use

What are we Measuring?

- Both buoyancy and elasticity are measures of RESPONSIVENESS
  - Of tax revenue to its underlying tax base
- Why two measures?
  - Buoyancy can be directly calculated from the data
    - Buoyancies do not correct for exogenous (discretionary) events such as policy changes
  - Elasticity can only be estimated from the data
    - Elasticities correct for exogenous (discretionary) events

Buoyancy

$$\text{Buoyancy}_t = \frac{\%\text{ change in tax revenue}_t}{\%\text{ change in proxy tax base}_t}$$

Interpretation: the observed percent change in revenue at time *t* compared to the observed percent change in the proxy tax base at time *t*, all included.

- Buoyancy reflects **all changes** in the tax system, including the tax rates and brackets, the definition of the base, variations in enforcement/compliance, or other specific shocks.

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Elasticity

$$\text{Elasticity} = \frac{\%\text{ change in tax revenue}}{\%\text{ change in proxy tax base}}$$

Interpretation: the percent change in tax revenue caused by a 1 percent change in the proxy tax base, keeping all else equal

- Elasticity assumes **no changes in policies,** that is changes in tax revenues mainly reflect changes in the tax base and not the effect of changes, for example, in statutory rates or compliance.

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Forecasting using buoyancy

	2011	2012	2013	2014	2015	2016
Import duties	97.0	100.8	103.8	114.2	126.0	140.7
Imports	1,000.0	1,050.0	1,070.0	1,120.0	1,200.0	1,300.0
% Change in Import duties	3.1	3.9	3.0	10.1	10.3	11.7
% Change in Imports	3.1	5.0	1.9	4.7	7.1	8.3
Buoyancy	1.0	0.8	1.6	2.2	1.4	1.4

Assume a buoyancy of 1.4 for 2016:

Projected % change in import duties: 8.3% * 1.4 = 11.7

Projected import duties: 126 * (1 + 11.7%) = 140.7

Rationale for the buoyancy approach

Very commonly used revenue forecasting method:

- Simple and transparent.
- Can be computed even with limited data.
- Very good to estimate effects of shocks to the proxy tax base.
- Can be applied to any level of tax (aggregate, or itemized).
- Can be used to compute "what if" analysis, especially to compute cyclically adjusted revenues.

Part 2. Interpreting elasticity and buoyancy

Interpretation

If…	the tax is…	which means that…
e > 1 (b > 1)	Elastic (Buoyant)	the tax revenue increases more than proportionately to a rise in the proxy tax base
e = 1 (b = 1)	Unitary Elastic (Buoyant)	the tax revenue increases proportionately to a rise in the proxy tax base
e < 1 (b < 1)	Inelastic (not buoyant)	the tax revenue increases less than proportionately to a rise in the proxy tax base

Elasticity, buoyancy, and the ETR (I)

Let T mean "tax revenue" and PB mean "proxy tax base"

$$\mathrm{e} = \frac{\dfrac{T_t - T_{t-1}}{T_{t-1}}}{\dfrac{PB_t - PB_{t-1}}{PB_{t-1}}} \qquad \mathrm{e} = \frac{\dfrac{T_t}{T_{t-1}} - 1}{\dfrac{PB_t}{PB_{t-1}} - 1}$$

If e = 1 ➡ $\dfrac{PB_t}{PB_{t-1}} - 1 = \dfrac{T_t}{T_{t-1}} - 1$ ➡ $\dfrac{T_{t-1}}{PB_{t-1}} = \dfrac{T_t}{PB_t}$

If e > 1 ➡ $\dfrac{T_t}{T_{t-1}} - 1 > \dfrac{PB_t}{PB_{t-1}} - 1$ ➡ $\dfrac{T_t}{PB_t} > \dfrac{T_{t-1}}{PB_{t-1}}$

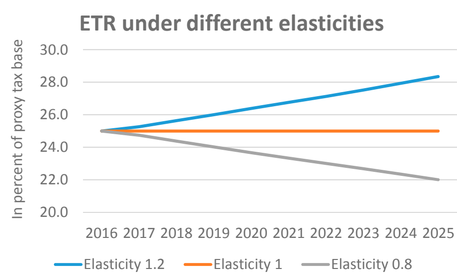
If e > 1 ➡ … < … ➡ … < …

Elasticity, buoyancy, and the ETR (II)

If e > 1 ➡ **ETR increases over time**

If e = 1 ➡ **ETR remains constant over time**

If e < 1 ➡ **ETR decreases over time**



Why can taxes be elastic or inelastic? (I)

Important ingredients:

- different rates applied to different values (subsets) of the tax base
- shifts (or composition changes) in the base
- delays in adjusting the rates
- the tax is a fixed amount
- collection lags in a context of high inflation
- delay in adjusting tax brackets with moderate inflation

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Why can taxes be elastic or inelastic? (II)

The effect of progressivity and base shift

From tax code			
Income brakets	< 60	60 - 100	>100
Statutory tax rate	15%	30%	50%

Income 2015	Person 1	Person 2	Person 3	Total
Taxable income	50	90	120	260
Applicable rate	15%	30%	50%	---
Personal tax	7.5	27	60	94.5

Income 2016	Person 1	Person 2	Person 3	Total
Taxable income	75	135	180	390
Applicable rate	30%	50%	50%	---
Personal tax	22.5	67.5	90	180

Percent change in tax:	90.5
Percent change in tax base:	50.0
Buoyancy:	**1.8**

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Why can taxes be elastic or inelastic? (III)

The effect of shifts in consumption patterns

From tax code		
Type of good	Services	Goods
Statutory tax rate	0%	20%

2015	Services	Goods	Total
Value	250	250	500
Applicable rate	0%	20%	---
Consumption tax	0	50	50

2016	Services	Goods	Total
Taxable income	350	300	650
Applicable rate	0%	20%	---
Personal tax	0	60	60

Percent change in tax:	20.0
Percent change in tax base:	30.0
Buoyancy:	**0.7**

Short and long run elasticity (I)

A tax can have an elasticity different from 1 in the short run; what would happen in the long run?

- taxes collected would either go to zero (e < 1), or…
- taxes would exceed the tax base (e > 1)

Over the long run, elasticity should converge to 1.

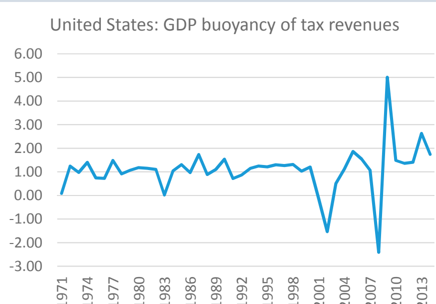
- Lag in indexation and distortions would be corrected
- Inflation would return to normal
- Base changes would stabilize (or tax code changed)
- Social pressure against high ETR

Part 3. Computing buoyancy and estimating the elasticity

Computing buoyancy

Buoyancy can be computed year by year using formula

$$\text{buoyancy}_t = \frac{\%\ \text{change in tax revenue}_t\ (T)}{\%\ \text{change in proxy tax base}_t\ (PB)}$$



Note how the buoyancy cycles with the economy. Should give caution in interpreting single year buoyancies, particularly in periods of economic volatility

Estimating elasticity

Elasticity is a measure of the response of taxes to the base **excluding the effects of policy changes**; it should be estimated with

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(an econometric technique) controlling for factors that capture policy changes.

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Pros and cons of elasticity approach

PROS	CONS
• Can be used for aggregates or single items • Simple to use • Best for scenario analysis with no policy change (analysis of shocks or baseline)	• Need to estimate the elasticity • Need to justify elasticity assumptions and differences between short / long run • Difficult to include effects of policy changes

- Note: for policy changes one needs micro models

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Estimating elasticity: step 0

STEP 0: take *log*T (call it *t*) and *log*PB (call it *pb*)

Consider what elasticity really means for how to model taxes:

$$T_t = \alpha(PB_t)^{\varepsilon}$$

Take log:

$$logT_t = \log\alpha + \varepsilon\, log(PB_t)$$

Take difference; if α is constant:

Elasticity (see Appendix)

$$\varepsilon = \frac{logT_t - logT_{t-1}}{log(PB_t) - log(PB_{t-1})}$$

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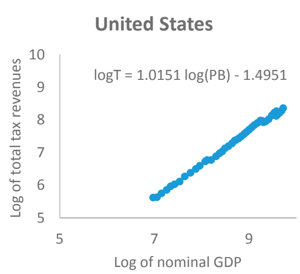
Simple estimate of elasticity

This generally works: estimate

STEP 1: Estimate

$$\boldsymbol{logT_t = \log\alpha + \varepsilon\, log(PB_t)}$$

by OLS



Strong assumption: all policy changes are somewhat random or they cancel each other out over time

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An Often Used Approach to Elasticity Analysis

- Estimate $$logT_t = \log \alpha + \varepsilon \, log(PB_t)$$
- Use ε as the elasticity
  - If ε is significantly different from 1, assume that in the long run ε will approach 1
- Why?
- Remember relationship with ETR

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Problems with this Approach

- Economic
  - Assertion that elasticity is 1 in the long run may not always be true
    - Behavioral changes over time may result in failure to approach 1
    - Under some circumstances, the elasticity may degrade and taxes may lose productivity
- Econometric
  - Estimating the simple log log elasticity equation in OLS will likely violate requirements for this estimation

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Perhaps a more appropriate method: Cointegration Analysis

If y and x are cointegrated then:

- The long run relationship is described by

$$y_t = c_2 + B_1 x_t$$

How y responds to x over the long run. When is B_1 an elasticity?

- In the short run, y adjusts to:
  - Past changes in x (and of y)
  - Past deviation from long run equilibrium

$$\Delta\, y_t = c_1 + \phi[y_{t-1} - c_2 - \varepsilon x_{t-1}] + \beta_1 \Delta x_{t-1} + \rho_1 \Delta\, y_{t-1}$$

How y responds to deviation from equilibrium

How y responds to x in the short run

Simple estimate of short run elasticity

Estimate the following:

$$\Delta\, t_t = c_1 + \phi[t_{t-1} - c_2 - \varepsilon pb_{t-1}] + \beta_1 \Delta pb_{t-1} + \rho_1 \Delta\, t_{t-1}$$

Note:

- The short run response of the tax to the base is β_1
- the proper estimation technique is Fully-Modified Least Squares on the equation above
- Unless we control for other factors which could affect taxes, we are assuming that policy changes cancel out, which could be a strong assumption. Better to control for other factors (tax rate, inflation, base shifts)

From estimates to forecasts

General rule to make things simple:
Use ε for the short run and 1 for the long run,
if not justify why

Note: if using β1 the full model should be used to forecast the tax, including the effect of the lag change of the tax and the adjustment to deviation from equilibrium. This implies that we must trust the model and its estimates!

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Implication of assumptions about elasticity

Remember that

- Elasticity > 1 implies that the ETR increases
- Elasticity = 1 implies that the ETR is constant
- Elasticity < 1 implies that the ETR decreases

Keep in mind that ETR could increase reflecting…

- Statutory rate increase
- Administrative improvements
- Base shift from low- to high- tax category

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Summary

Conclusion

- No best forecasting methods: use both elasticity and ETR and see what the assumption about one implies for the other
- Elasticity should be estimated, but this can be safely done with simple OLS
- Remember the simple math of elasticity and ETR: if the elasticity is >1 then ETR increases, if it is =1 the ETR remains constant, and if the elasticity <1 the ETR declines.

Appendix

Taking log to obtain elasticity

Consider $$\varepsilon = \frac{logT_t - logT_{t-1}}{log(PB_t) - log(PB_{t-1})}$$

Remember, the difference of a log is the log of the ratio

$$\varepsilon = \frac{log\left(\frac{T_t}{T_{t-1}}\right)}{log\left(\frac{PB_t}{PB_{t-1}}\right)} \quad \text{or} \quad \varepsilon = \frac{log\left(\frac{T_t - T_{t-1}}{T_{t-1}} + 1\right)}{log\left(\frac{PB_t - PB_{t-1}}{PB_{t-1}} + 1\right)}$$

Remember, log of 1 plus a small x is about x

$$\varepsilon = \frac{\frac{T_t - T_{t-1}}{T_{t-1}}}{\frac{PB_t - PB_{t-1}}{PB_{t-1}}}$$

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L12 – Overall balance, interest payment, net lending/borrowing and debt

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Outline of the lecture

1. An Introduction to Fiscal Accounts
2. Forecasting Interest Payments, Borrowing and Debt
3. Debt Dynamics
4. Ways of Budget Financing
5. Macroeconomic implications of budget financing
6. Fiscal implications of macroeconomic developments

I. Fiscal Accounts

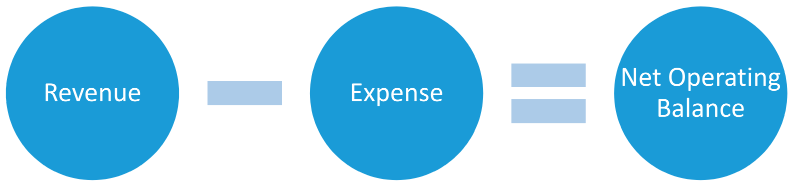
3

I. Fiscal Accounts

The Statement of Government Operations – Above the Line Transactions

I.	*Transactions affecting net worth*		
1.	Revenue (increase net worth)		
2.	Expense (reduce net worth)		
3.	**Gross Operating Balance**		**= 1 – 2**
	2.8	Consumption of fixed capital (depreciation)	
4.	**Net Operating Balance ➡ Change in net worth**		**= 3 – 2.8**
II.	*Transactions in nonfinancial assets*		
	5.1	Net acquisition of nonfinancial assets	
6.	**Overall Balance = Net lending/Borrowing**		**= 4 – 5.1**

I. Fiscal Accounts
Net operating balance



• Net operating balance represents the change in net worth due to transactions attributable to government policy.

• Focus: the "activity" side of government (by considering transactions in revenue and expense)

• With a positive balance, government can acquire assets or decrease liabilities.

•With a negative balance, the government must incur liabilities or liquidate assets.

I. Fiscal Accounts
A. Net lending and borrowing

• ***Net Lending/Borrowing*** *(NL/B)* is a summary measure indicating the extent to which government is either putting financial resources at the disposal of other sectors in the economy or using the financial resources generated by other sectors.

• Focus: on the "financing" side (by considering transactions in financial assets and liabilities).

• NL/B as an indicator of the financial impact of government activity on the rest of the economy.

I. Fiscal Accounts

The Statement of Government Operations – Above and Below the Line Transactions

I. Transactions affecting net worth			
1.	Revenue (increase net worth)		
2.	Expense (reduce net worth)		
3.	**Gross Operating Balance**		**= 1 – 2**
2.8	Consumption of fixed capital (depreciation)		
4.	**Net Operating Balance ➨ Change in net worth**		**= 3 – 2.8**
II. Transactions in nonfinancial assets			
5.1	Net acquisition of nonfinancial assets		
6.	**Net lending/Borrowing**		**= 4 – 5.1**
III. Transactions in financial assets and liabilities			= 5.2 + 5.3 = – 6
5.2	Net acquisition of financial assets		
5.3	Net incurrence of liabilities		
5.3.1		Foreign	
5.3.2		Domestic	

I. Fiscal Accounts

"Above" and "Below" the line

- **"the line"** refers to whether an item is recorded as part of the overall balance, or instead as financing
- **Revenue** (taxes, profits, and grants) are recorded "above the line"
- **Expense and acquisition of non-financial asset** are "above the line"
- **Repayable resources** (borrowing from markets or from international financial institutions) are treated as financing and are "below the line".

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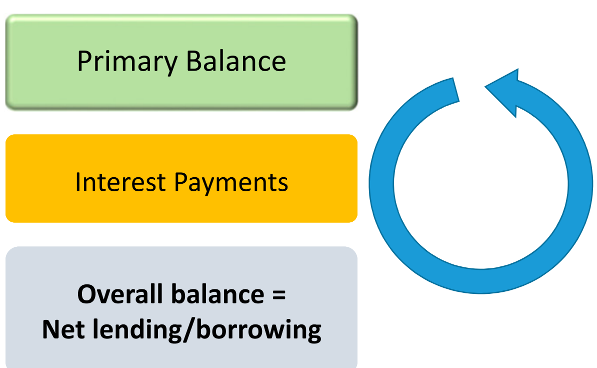
II. Forecasting interest payments, borrowing and debt

II. Forecasting interest payments, borrowing and debt

The Simultaneity Problem



Simultaneity problem: solve with iterative process:

1. Make an initial estimate of interest payments
2. Compute overall balance and new borrowing requirements
3. Recalculate interest payments and start again

II. Forecasting interest payments, borrowing and debt
A simple example

Ideally, forecast interest payment on each new issuance separately.

	2014 Q1	2014 Q2	2014 Q3	2014 Q4	2015 Q1	2015 Q2
Disbursement (4% interest, quarterly payments)	100					
Interest Payment		1	1	1	1	
Disbursement (10% interest, semi-annual payments)		140				
Interest Payment			0	7	0	7
Total Interest payment				8		

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II. Forecasting interest payments, borrowing and debt
Example 1

- **Debt at end 2013 = 1,000; Interest rate on debt = 8% (paid quarterly)**
- **Primary Balance = -200 (equally divided every quarter)**

	2014Q1	2014Q2	2014Q3	2014Q4	2014
Round 1					
Primary Balance	-50	-50	-50	-50	-200
Interest on old debt	20	20	20	20	80
Overall Balance	-70	-70	-70	-70	-280
Gross Borrowing	70	70	70	70	280
Debt stock					**1280**
Round 2					
Interest on new debt (10%)	0	1.75	3.5	5.25	10.5
New Overall Balance	-70	-71.75	-73.5	-75.25	-290.5
Additional Borrowing	0	1.75	3.5	5.25	10.5
Revised Debt Stock					**1290.5**
Round 3					
Interest on new debt (10%)	0	0	0.044	0.131	0.175
New Overall Balance	-70.00	-71.75	-73.54	-75.38	-290.68
Additional Borrowing	0	0	0.044	0.131	0.175
Revised Debt Stock					**1290.675**

II. Forecasting interest payments, borrowing and debt

Example 2: Including amortization

- Now assume, of the old debt stock, that 100 is amortized with the proceeds of a public asset) in 2014Q2

	2014Q1	2014Q2	2014Q3	2014Q4	2014
Round 1					
Primary Balance	-50	-50	-50	-50	-200
Interest on old debt	20	20	18	18	76
Overall Balance	-70	-70	-68	-68	-276
Gross Borrowing	70	70	68	68	276
Debt stock					**1176**
Round 2					
Interest on new debt (10%)	0	1.75	3.5	5.2	10.45
New Overall Balance	-70	-71.75	-71.5	-73.2	-286.45
Additional Borrowing	0	1.75	3.5	5.2	10.45
Revised Debt Stock					**1186.45**
Round 3					
Interest on new debt (10%)	0	0	0.044	0.131	0.175
New Overall Balance	-70.00	-71.75	-71.54	-73.33	-286.63
Additional Borrowing	0	0	0.044	0.131	0.175
Revised Debt Stock					**1186.625**

II. Forecasting interest payments, borrowing and debt

Example 3: Including amortization with new issuance

- Now assume, of the old debt stock, that 100 is amortized in 2014Q2 through the issuance of new debt with interest rate of 10%

	2014Q1	2014Q2	2014Q3	2014Q4	2014
Round 1					
Primary Balance	-50	-50	-50	-50	-200
Interest on old debt	20	20	20.5	20.5	81
Overall Balance	-70	-70	-70.5	-70.5	-281
Gross Borrowing	70	70	70.5	70.5	281
Debt stock					**1281**
Round 2					
Interest on new debt (10%)	0	1.75	3.5	5.26	10.51
New Overall Balance	-70	-71.75	-74	-75.76	-291.51
Additional Borrowing	0	1.75	3.5	5.26	10.51
Revised Debt Stock					**1291.51**
Round 3					
Interest on new debt (10%)	0	0	0.044	0.131	0.18
New Overall Balance	-70.00	-71.75	-74.04	-75.89	-291.69
Additional Borrowing	0	0	0.044	0.131	0.18
Revised Debt Stock					**1291.69**

II. Forecasting interest payments, borrowing and debt
A simpler method

A valid shortcut consists of assuming that new borrowing is incurred at mid-year (Denote: I = Total Interest Payment; i is interest rate)

$$I_t = i_t^{old} D_{t-1} + i_t^{new} \frac{\text{new borrowing}}{2}$$

- If $i_t^{old} \neq i_t^{new}$ compute the "implicit" interest rate so that:

$$I_t = i_t^* \left[D_{t-1} + \frac{D_t - D_{t-1}}{2} \right] = i_t^* \left[\frac{D_t + D_{t-1}}{2} \right]$$

- For I_{t+1}, we need to make projections of the future implicit interest rate.

II. Forecasting interest payments, borrowing and debt
Forecasting interest rate on future borrowing

Steps:

1. Check historical trend of past interest rate on government debt relative to some benchmark (central bank policy rate, international rate—LIBOR, US T-bill rate…)
2. Consider if there is likely to be an change in the risk premium (spread over the benchmark) on future debt issuances
3. Make projections of the benchmark rate and the risk premium, and hence the future rate on government debt

The inertia of future implicit interest rate depends on the amount of debt maturing next year

II. Forecasting interest payments, borrowing and debt
Forecasting implicit interest rate – example 1

	2014	2015
Stock of debt outstanding at year end	1000	1200
Amount maturing in mid-2016	…	200
New borrowing	…	400
Implicit interest rate on old debt stock	**5%**	…
Interest payment on old debt stock	…	45 = 5%*800 + 2.5%*200
Interest rate on new borrowing	…	20%
Interest payment on new borrowing	…	40 = 10%*400
Total interest payments	…	95
Average stock of debt	…	1100
Implicit interest rate on new debt stock	…	**8.6%**

II. Forecasting interest payments, borrowing and debt
Forecasting implicit interest rate – example 2

	2014	2015
Stock of debt outstanding at year end	1000	1200
Amount maturing in mid-2015	…	800
New borrowing	…	1400
Interest rate on old debt stock	5%	…
Interest payment on old debt stock	…	20 = 5%*200 + 2.5%*800
Interest rate on new borrowing	…	20%
Interest payment on new borrowing	…	140 = 10%*1400
Total interest payments	…	160
Average stock of debt	…	1100
Implicit interest rate on new debt stock	…	14.5%

III. Debt Dynamics

III. Debt Dynamics
Accumulation of New Debt

$$\text{Public debt}_t - \text{Public debt}_{t-1} = \underbrace{-\text{Net lending/borrowing}}_{\text{Overall balance}} + \underbrace{\text{other flows}}_{\text{E.g. Privatization receipts}}$$

$$\text{Public debt}_t - \text{Public debt}_{t-1} = \text{Interest rate x Public Debt}_t - \text{Primary balance (PB)} + \text{Other Flows}$$

II. Public Debt Dynamics

What are the "other flows"?

Flows reducing debt stock

- Exceptional Financing: Debt reduction (MDRI, bilateral debt relief)
- Privatization receipts
- Asset valuation (exchange rate effects)

Flows increasing debt stock

- Contingent liabilities
- Asset valuation (exchange rate effects)

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III. Public Debt Dynamics

Notation (ignoring external debt)

D_t stock of debt at time t

PB_t primary or non-interest surplus at time t

O_t Other Flows at time t $\quad \Delta D_t = iD_{t-1} - PB_t + O_t$

r_t real interest rate at time t

π_t inflation rate at time t

i_t nominal interest rate at time t $\quad 1 + i_t = (1 + r_t)(1 + \pi_t)$

g_t real GDP growth rate at time t

$P_t Y_t$ nominal GDP at time t $\quad P_t Y_t = (1 + \pi_t)(1 + g_t)P_{t-1}Y_{t-1}$

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III. Public Debt Dynamics
Law of motion of debt and ignoring other flows

- Government Debt at time t $$D_t = (1+i_t)D_{t-1} - PB_t$$
- Divide equation above by P_tY_t (nominal GDP):

$$\underbrace{\frac{D_t}{P_tY_t}}_{d_t} = \underbrace{\frac{(1+i_t)}{(1+\pi_t)(1+g_t)}}_{\phi_t} \underbrace{\frac{D_{t-1}}{P_{t-1}Y_{t-1}}}_{d_{t-1}} - \underbrace{\frac{PB_t}{P_tY_t}}_{pb_t}$$

where π is inflation; g is real GDP growth

- Note that: where r is the real interest rate

$$\phi_t = \frac{(1+i_t)}{(1+\pi_t)(1+g_t)} = \frac{(1+r_t)}{(1+g_t)}$$

$$d_t = \phi_t d_{t-1} - pb_t$$

Key equation

If $\phi_t < 1$ (i.e. $r_t < g_t$) then d_t converges, and debt is sustainable
If $\phi_t > 1$ (i.e. $r_t > g_t$) , then d_t explodes

IV. Ways of financing the budget

Financing the budget: forms

The way in which the deficit is financed has macroeconomic consequences.

There are four forms of financing:

- Borrowing from the central bank
- Borrowing from the domestic commercial banks
- Borrowing from the domestic nonbank sector
- Borrowing abroad

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Financing the budget

Total financing = - overall balance
Foreign (net)
New borrowing
Repayments
Domestic
Bank lending (net) – including central bank
New borrowing
Repayments of debt
Deposits
Non-Bank (net)
Privatization receipts

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Financing the budget

- Deposits at the central bank are an asset: accumulating deposits is an increase in assets (= decrease in net debt), drawing-down on deposits is a decrease in assets (= increase in net debt)
- Borrowing abroad (or domestically) to increase deposits is neutral on net debt (why?)

V. Macro implications of budget financing

Financing the budget

- Central bank borrowing: by expanding money supply, can add to inflation. But monetary expansion can also increase the impact of fiscal expansion on demand.
- Commercial bank borrowing: crowds out private borrowing, raising interest rates, unless central bank accommodates by supplying reserves. Then like above.
- Domestic non-bank borrowing: crowds out private borrowing that could be used to finance investment, less funding available for loans leads to higher interest rates
- Foreign borrowing: raises foreign debt and may lead to Balance of Payment problems; exchange rate risk; debt service needs may exert downward pressure on the exchange rate

Financing the budget

The type and amount of public debt has also consequences on:

- Macroeconomic environment
- Policy space
- Financial sector stability

Financing the Budget: Balance sheet connections across sectors (I)

- Consider the stock of assets and liabilities at a certain point in time for the main sectors of the economy:
  - the government (including the central bank)
  - the private financial sector (mainly banks)
  - the non-financial sector (corporations and households)
  - external (non-resident) entities

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Financing the Budget Balance sheet connections across sectors (II)

Government sector	
Assets	**Liabilities**
Claims on: Financial sector Non-fin private sector External	**Obligations to:** Financial sector Non-fin private sector External **Net worth**

Financial sector	
Assets	**Liabilities**
Claims on: Government sector Non-fin private sector External	**Obligations to:** Government sector Non-fin private sector External **Net worth**

Non-financial private sector	
Assets	**Liabilities**
Claims on: Government Financial sector External	**Obligations to:** Government Financial sector External **Net worth**

External sector	
Assets	**Liabilities**
Claims on: Government Financial sector Non-fin private sector	**Obligations to:** Government Financial sector Non-fin private sector **Net worth**

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VI. Fiscal effects of macroeconomic developments

Real GDP and the budget

- During an economic contraction revenues-to-GDP fall and expenditure-to-GDP increases
- Despite high growth, fall in commodity prices can cause revenues-to-GDP to fall
- In good times higher revenues may *trigger higher discretionary* expenditures i.e. induce pro-cyclicality and allocation of expenditures may not be optimal

Inflation and the budget

- Increase in commodity prices and cost of subsidies
- Higher inflation rate could lead to higher nominal interest rates and higher interest spending in government budget
- Higher inflation rate would lead to appreciation of currency in real terms and loss of competitiveness
- Very high inflation could lead to uncertainty, decline in investments, and thus GDP and revenues

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Real interest rates and the budget

- Lower interest rates mean lower interest payments
- Lower interest rates could spur investment and higher GDP growth; also corporate costs decline leading to higher corporate profits and tax collections
- Low foreign interest rates – search for yield could lead to capital inflows
- High capital inflows could lead to more liquidity supporting higher growth as well as lower interest rates (maybe higher inflation) and higher revenues

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Real exchange rate and the budget

- If the Real Effective Exchange Rate (REER) appreciates, lose competitiveness lowering GDP:
  - Exports decline
  - Cheaper imports hurt domestic industry
- High external debt – if exchange rate depreciates or global interest rates rise, liability increases as do interest payments; if not rolled over could cause capital flight and lower growth and revenues
- Real appreciation may lead to capital flows and higher revenues on capital gains

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Development, demographics and the budget

- As an economy develops the government can rely on a more diverse tax base (adoption of technology makes it easier to comply, to better target beneficiaries of exemption, to improve tax administration…)
- Richer economies can usually support more extensive social security and safety nets and have a larger government
- Age structure affects mix of expenditure (if young population, need to spend more on their education; an aging population requires more spending on health care and public pension; also, revenue growth may also be declining with large old population)

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Thank you

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Workshop on Macro-Fiscal Forecasting
Arusha, August 8 – 18, 2016

L13 – Analysis of Fiscal Aggregates, Adjusted Balances, and Fiscal Position

Fazeer Rahim (IMF East AFRITAC)

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Outline of the lecture

1. Fiscal Aggregates
2. Adjusted Balances
3. Fiscal Position

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I. Fiscal Aggregates

Introduction

- Activities of the government sector
- Collect revenue
- Produce goods and services
- Redistribute income via transfers

- Activities lead to financing to and from the rest of the economy.

Questions:
What is the government's macroeconomic impact?
Is government activity sustainable over time?

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Introduction
Why use fiscal indicators

- Fiscal indicators shed light on:
  - How does the fiscal sector impact domestic demand?
    - Expenditure: changes in expenditure affect aggregate demand
    - Revenue: changes in revenue affect aggregate demand
  - What is the stance of fiscal policy?
    - What is the underlying fiscal position?
    - Is discretionary fiscal policy expansionary or contractionary?

  - Is public debt sustainable?

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I. Fiscal Aggregates
Different uses of fiscal indicators

I. Fiscal Accounts – Statement of Government Operations

I.	*Transactions affecting net worth*			
1.	Revenue (increase net worth)			
2.	Expense (reduce net worth)			
3.	**Gross Operating Balance**			**= 1 – 2**
	2.8	Consumption of fixed capital (depreciation)		
4.	**Net Operating Balance ➡ Change in net worth**			**= 3 – 2.8**
II.	*Transactions in nonfinancial assets*			
	5.1	Net acquisition of nonfinancial assets		
		5.1.1	Fixed assets	
		5.1.2	Change in inventories	
		5.1.3	Other	
6.	**Net lending/Borrowing**			**= 4 – 5.1**
III.	*Transactions in financial assets and liabilities*			**= 5.2 + 5.3 = – 6**
	5.2	Net acquisition of financial assets		
	5.3	Net incurrence of liabilities		
		5.3.1	Foreign	
		5.3.2	Domestic	

I. Fiscal Aggregates
Net operating balance

• Net operating balance represents the change in net worth due to transactions attributable to government policy.

• Focus: the "activity" side of government (by considering transactions in revenue and expense)

• With a positive balance, government can acquire assets (financial or non-financial) or decrease liabilities.

•With a negative balance, the government must incur liabilities or liquidate assets (financial or non-financial)

I. Fiscal Aggregates
Net lending and borrowing

• ***Net Lending/Borrowing*** *(NL/B)* is a summary measure indicating the extent to which government is either putting financial resources at the disposal of other sectors in the economy or using the financial resources generated by other sectors.

• Focus: on the "financing" side (by considering transactions in financial assets and liabilities).

• NL/B as an indicator of the financial impact of government activity on the rest of the economy.

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I. Fiscal Aggregates
Primary balance

Definition: Primary balance = overall balance + interest payments

Rationale:
•An indicator of current fiscal effort, since interest payments are predetermined by the size of previous deficits

•The primary balance is a key variable used in debt-sustainability analysis.

I. Fiscal Aggregates
Non-resource balances

- Resource revenue: Revenue receivable that is related to natural resources. These receivables may be related to various types of taxes, subsidies, dividends, contracts, leases, and licenses, rent, or other transfers.
- Resource expense: Expense payable that is related to natural resources. These payables may be related to various types of expense such as subsidies, property expense, and transfers.

• Non-resource balance: overall balance excluding net resource revenue. A measure of fiscal stance

• Non-resource primary balance: primary balance excluding net resource revenue. A measure of fiscal sustainability

Rationale: resource revenue are volatile and exhaustible

II. Adjusted Fiscal Balances

II. Adjusted Fiscal Balances
Cyclically-adjusted balance

Definition: The cyclically-adjusted balance (CAB) "strips" the cyclical effects from the overall balance. Both revenues and expenditures (e.g. income and sales tax revenues, unemployment insurance outlays) are adjusted.

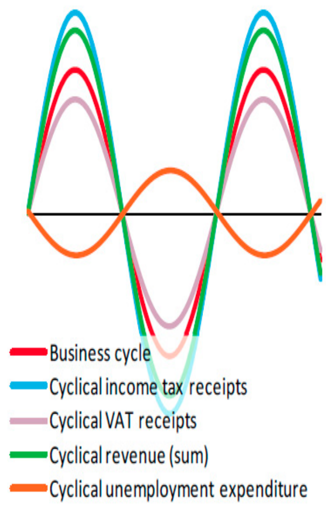
Cyclical adjustment corrects for variations in fiscal revenue and expenditure due to the business cycle.

Rationale: We should adjust the overall fiscal balance to account for factors that cannot be imputed to government action, but are attributable to business cycle

II. Adjusted Fiscal Balances
Why cyclical adjustment?

- Revenue, expense and overall balance could be misleading measures of the fiscal policy stance
- Some fiscal variables respond automatically to temporary changes in output.
- For example, when the output gap is negative:
  - Unemployment benefits are higher than usual.
  - Household and corporate incomes are lower and, therefore, most tax revenues are lower. Same goes for HH consumption and VAT.
  - Revenues from taxes on capital gains and property are also likely to be lower.
  - If the tax system is progressive, tax revenues will be lower still.

→ automatic stabilizers



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II. Adjusted Fiscal Balances
Cyclically-adjusted balance: the Importance of Automatic Stabilizers

- The most important factor determining the cyclical sensitivity of the fiscal position is the *size of the general government sector.*
- Tax structure is also important: The greater the taxation of cyclically-sensitive tax bases, the more cyclical revenues will be.
- Other factors: the generosity of transfer incomes and the progressivity of the tax system.

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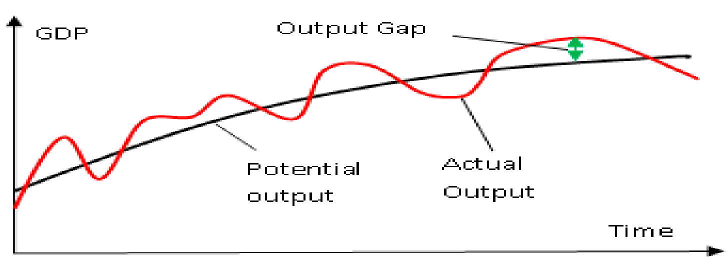
II. Adjusted Fiscal Balances
Procedure for cyclical adjustment

CAB = Overall balance – cyclical component of the balance

Obtaining cyclical component requires:
- An estimate of the **output gap**
- **Elasticities** of revenue/expenditure to the output gap

II. Adjusted Fiscal Balances
Output Gap

- Difference between actual and potential output (a measure of **business cycles**)



- Often expressed as percentage of potential output

$$\text{Output gap} = \frac{Y - Y^*}{Y^*}$$

II. Adjusted Fiscal Balances

Estimating Potential Output

- Time series techniques (IMF offers MF course):
  - Linear regression
  - **Hodrick-Prescott (HP) filter**
- Production function approach
- There is **no formal test** that separates good estimates from bad ones

Reference: [EC Economic Papers 247](), [IMF African Dept. Paper 11/02](), and others

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II. Adjusted Fiscal Balances

Revenue and the Cycle

Actual GDP

Nominal revenues (total or itemized)

Elasticity

$$R_i = R_i^* \left(\frac{Y}{Y^*} \right)^{\varepsilon_i}$$

Is ε_i smaller or greater than zero?

$\varepsilon_i > 0$: revenues are procyclical

Adjusted revenues: nominal revenues (total or itemized) that would prevail with no output gap

Potential GDP

Note: Here, (Y/Y*) represents the output gap.

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II. Adjusted Fiscal Balances

Expenditure and the Cycle

Actual GDP

Nominal expenditures
(total or itemized)

Elasticity

$$G_i = G_i^* \left(\frac{Y}{Y^*} \right)^{\eta_i}$$

Is η_i smaller or greater than zero?

$\eta_i \leq 0$: expenditures are counter-cyclical

Adjusted expenditures:
Nominal expenditures (total or itemized) that would prevail with no output gap

Potential GDP

Note: Here, (Y/Y*) represents the output gap.

II. Adjusted Fiscal Balances

Computing the CAB

Step 1: Compute the revenues and expenditures that would have prevailed if output had been at potential

$$R_i^* = R_i \left(\frac{Y^*}{Y} \right)^{\varepsilon_i} \qquad G_i^* = G_i \left(\frac{Y^*}{Y} \right)^{\eta_i}$$

Step 1.b: If itemized, sum to obtain total adjusted revenues and expenditures

Step 2: Compute the CAB in percent of potential output

$$cab^* = \frac{R^* - G^*}{Y^*}$$

Revenue and expenditure items excluded from the adjustment (for example, interest payments)

II. Adjusted Fiscal Balances
CAB and Actual Overall Balance

Recall that:

$$R_i^* = R_i \left(\frac{Y^*}{Y}\right)^{\varepsilon_i} \quad \varepsilon_i > 0 \qquad G_i^* = G_i \left(\frac{Y^*}{Y}\right)^{\eta_i} \quad \eta_i \le 0$$

Boom:

$$Y > Y^* \Rightarrow R > R^*, G \le G^*$$

CAB < Actual Balance

Public finances (actual b) are in better shape than they would be (CAB) because economy is in a boom.

Recession:

$$Y < Y^* \Rightarrow R^* > R, G^* \le G$$

CAB > Actual Balance

Public finances would have been in better shape, if the economy were not in a recession.

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II. Adjusted Fiscal Balances
CAB: Aggregated vs. Disaggregated Approach

- **Aggregate approach:**
  - Same elasticity for all revenues and same elasticities for all expenditures elasticities can be
  - common values are 1 for revenue and 0 for expenditure
  - Source: from the literature or estimated
- Disaggregated approach (OECD methodology):
  - Consider different elasticities for **disaggregated** categories of revenue and expenditure
    - Corporate income tax
    - Personal income tax
    - Indirect taxes (VAT, import duties, excises)
    - Social security contributions

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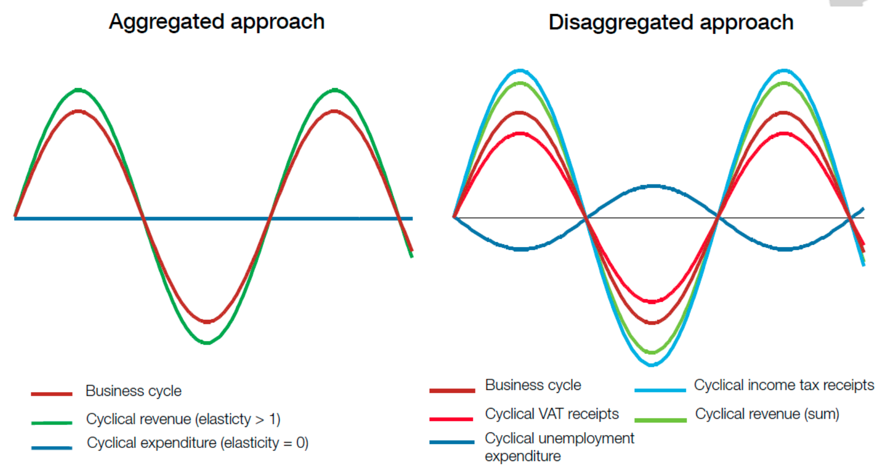
II. Adjusted Fiscal Balances

CAB: Aggregated vs. Disaggregated Approach



Reference: IMF Technical Note and Manuals 11/02

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II. Adjusted Fiscal Balances

Disaggregated Elasticities – some country examples

TABLE 2. OECD METHODOLOGY: SUMMARY OF ELASTICITIES[1]

(Elasticities with respect to the output gap)

	Corporate Tax	Personal Tax	Indirect Taxes	Social Security Contributions	Primary Current Expenditure	Overall Balance[2]
	η_1	η_2	η_3	η_4	κ	
United States	1.53	1.30	1.00	0.64	–0.09	0.34
Japan	1.65	1.17	1.00	0.55	–0.05	0.33
Germany	1.53	1.61	1.00	0.57	–0.18	0.51
France	1.59	1.18	1.00	0.79	–0.11	0.53
Italy	1.12	1.75	1.00	0.86	–0.04	0.53
United Kingdom	1.66	1.18	1.00	0.91	–0.05	0.45
Canada	1.55	1.10	1.00	0.56	–0.12	0.38
Australia	1.45	1.04	1.00	0.00	–0.16	0.39
Austria	1.69	1.31	1.00	0.58	–0.08	0.47
Belgium	1.57	1.09	1.00	0.80	–0.14	0.52
Czech Republic	1.39	1.19	1.00	0.80	–0.02	0.39
Denmark	1.65	0.96	1.00	0.72	–0.21	0.59
Finland	1.64	0.91	1.00	0.62	–0.18	0.48
Greece	1.08	1.80	1.00	0.85	–0.04	0.47
Hungary	1.44	1.70	1.00	0.63	–0.03	0.47
Iceland	2.08	0.86	1.00	0.60	–0.02	0.37
Ireland	1.30	1.44	1.00	0.88	–0.11	0.38
Korea	1.52	1.40	1.00	0.51	–0.04	0.22

TABLE 2. OECD METHODOLOGY: SUMMARY OF ELASTICITIES[1]

(Elasticities with respect to the output gap)

	Corporate Tax	Personal Tax	Indirect Taxes	Social Security Contributions	Primary Current Expenditure	Overall Balance[2]
	η_1	η_2	η_3	η_4	κ	
Luxembourg	1.75	1.50	1.00	0.76	–0.02	0.47
Netherlands	1.52	1.69	1.00	0.56	–0.23	0.53
NewZealand	1.37	0.92	1.00	0.00	–0.15	0.37
Norway (mainland)	1.42	1.02	1.00	0.80	–0.05	0.53
Poland	1.39	1.00	1.00	0.69	–0.14	0.44
Portugal	1.17	1.53	1.00	0.92	–0.05	0.46
Slovak Republic	1.32	0.70	1.00	0.70	–0.06	0.37
Spain	1.15	1.92	1.00	0.68	–0.15	0.44
Sweden	1.78	0.92	1.00	0.72	–0.15	0.55
Switzerland	1.78	1.10	1.00	0.69	–0.19	0.37
OECD average	1.50	1.26	1.00	0.71	–0.10	0.44
Euro area average	1.43	1.48	1.00	0.74	–0.11	0.48
New EU members average	1.38	1.15	1.00	0.71	–0.06	0.42

Source: Girouard et al. (2005), page 22.
[1]Based on 2003 weights. Averages are unweighted.
[2]Semi-elasticity. It measures the change of the budget balance, as percentage of GDP, for a 1% change in GDP.
Share weighting based on 2003.

- EC uses these estimates in their budgetary surveillance (EC Paper (2005))

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II. Adjusted Fiscal Balances

Disaggregated Elasticities – how to calculate your own

- In general,

$$\varepsilon_i = \varepsilon_{i,\text{tax base } i}\ \varepsilon_{\text{tax base } i, \frac{Y}{Y^*}}$$

Elasticity of tax revenues to its tax base

Elasticity of tax base to output gap

How to calculate the elasticities:

1. Tax revenue to its tax base: regress the log of tax base on log of tax revenue
2. Tax base to output: regress the log of output gap on log of tax base

TABLE 1. COMMON TAX ELASTICITIES

Tax category	Elasticity of tax revenue relative to its base	Elasticity of base relative to output gap	Elasticity of tax revenue relative to output gap
Personal income taxes	≈ 1.5–2	≈ 0.6–0.9	≈ 1.0–1.7
Corporate income taxes	≈ 1	≈ 1.2–1.8	≈ 1.2–1.8
Social security contributions	≈ 0.8–1.1	≈ 0.6–0.9	≈ 0.5–0.9
Indirect taxes	≈ 1	≈ 1	≈ 1

Source: Girouard and André (2005).

II. Adjusted Fiscal Balances

CAB: Finding elasticities - evidence from sub-Saharan Africa

- Elasticity of revenue to GDP
  - Revenue = Taxes + Non-tax revenue + Grants
    - Most taxes are in the form of indirect taxes. Indirect taxes are pro-cyclical, with elasticity of revenue to potential GDP is close to 1
    - Non-tax revenue are small relative to total revenue
    - Foreign grants are important in many countries and they tend to be acyclical, i.e. elasticity close to 0.
- Elasticity of expenditure to GDP
  - Expenditure = current + capital + transfer payment
    - Most of expenditures are not related to transfer payments
    - Current and capital expenditure are acyclical, i.e. elasticity close to 0.

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II. Adjusted Fiscal Balances
Structural balance

- Structural adjustment also corrects for the impact of one-off operations, and other transient influences (asset and commodity prices, output composition effects, etc.) beyond the output cycle.
- The **structural balance facilitates the separation of fiscal balances into discretionary and non-discretionary parts**

Rationale: We need to adjust the fiscal balance to account for factors that cannot be imputed to government action, but can be attributable to business cycle, commodity price shocks and other one-off events

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II. Adjusted Fiscal Balances
Structural Balance - Steps

Structural balance is the overall balance adjusted for a broader range of exogenous factors.
It is broader than the CAB as it includes one-off factors and other asset price changes (e.g. commodity prices)

Procedure (3 steps)

Step 1: Identify and remove one-off fiscal operations

Step 2: Adjust for effects of business cycles

Step 3: Adjust for effects of other factors (asset prices, etc.)

- Can be viewed as an **augmentation of CAB**
- Can be a better measure of **underlying fiscal balance** if effects of other cycles and factors are significant

III. Fiscal Position

III. Fiscal Position
Fiscal Stance

Quantifies how much fiscal policy add to or subtract from **domestic demand**

- Expansionary (or loose)
- Contractionary (or tight)

Definition, using CAB: $FS_t = -CAB_t$

$CAB_t < 0 \Longrightarrow$ Expansionary ($FS_t > 0$)

$CAB_t = 0 \Longrightarrow$ Neutral ($FS_t > 0$)

$CAB_t > 0 \Longrightarrow$ Contractionary ($FS_t < 0$)

- Note: We can use other fiscal indicators to measure fiscal stance. E.g. Non-resource primary balance or structural balance

III. Fiscal Position
Fiscal Impulse

Fiscal impulse measures the change in fiscal stance over time:

Fiscal impulse: $FI = FS_t - FS_{t-1}$

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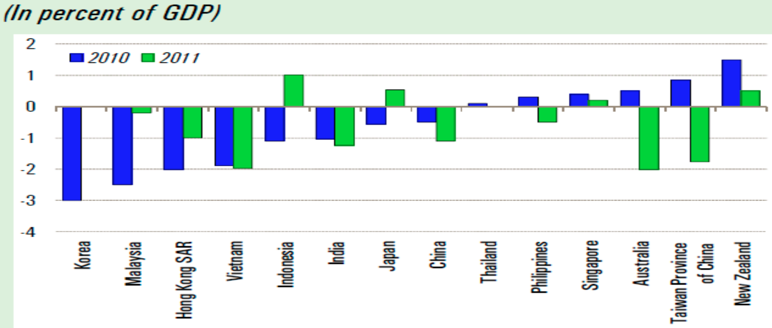
III. Fiscal Position
The fiscal impulse – an example

Question: how has the fiscal impulse changed from 2011 to 2010 in Asia?

Figure 1.9. Asia: Fiscal Impulse[1]
(In percent of GDP)



Source: IMF staff estimates.
[1] Based on annual change (calendar year) in general government cyclically adjusted fiscal balance-to-GDP ratios. A negative number implies withdrawal of fiscal stimulus.

Source: REO APD Apr 2011 72

III. Fiscal Position
Likely response to a fiscal impulse

Does an expansionary fiscal impulse actually lead to an economic expansion?

It depends on:

- **Expectations** by the private sector
- How the deficit is **financed**
  e.g. temporary stimulus followed by an increase in tax
- **Composition** of the change in fiscal policy
  Note: Fiscal stance and impulse measures implicitly assume that the impact of different instruments are the same.

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